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Variance Estimation Using Linear Combination of Standard Deviation and Deciles

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Authors' contributions

This work was carried out in collaboration between all authors. Author MAB designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author SM managed the analysis of the study. Authors AR, SAS and MAM managed the literature searches . All authors read and approved the final manuscript.

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ABSTRACT

In this present study, we have introduced a new type of ratio estimators for the estimation of finite population variance using standard deviation and deciles as auxiliary variables to improve the efficiency of proposed estimators. A comparison between suggested estimators and existing estimators has been made through a numerical illustration to prove the efficiency of proposed estimators over existing estimators. The expression for bias and mean square error has been derived up to the first order of approximation. The improvement of proposed estimators over existing estimators shown is clearly based on the lesser mean square error of proposed estimators.

Keywords: Simple random sampling; bias; mean square error; standard deviation and deciles; efficiency.

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1. INTRODUCTION

The strategy of modifying the estimators through proper utilization of auxiliary information has been widely discussed by different authors in different forms when there exists a close association between auxiliary variable (X) and Study variable (Y). Several authors have proposed different estimators to improve the efficiency of estimators. Some of them from the literature are such as Isaki [1], who proposed ratio and regression estimators. Upadhyaya and Singh [2] used coefficient of kurtosis β_{2x} as auxiliary variable to improve the efficiency of estimator. Kadilar & Cingi, (H.2006a) [3] utilized the coefficient of skewness C_x as auxiliary population parameter to enhance the efficiency of estimator. Subramani. j and Kumarapandiyan .G [4] used quartiles as auxiliary information to improve the efficiency of modified estimators over existing estimators. On the same lines, Singh. D, and Chaudhary, F.S [5], M .Murthy [6], Arcos. A . M. Rueda, M. D, Martinez. S . Gonzalez and Y. Roman [7], have utilized this auxiliary information in different forms to enhance the precision and efficiency of proposed estimators. Recently, Subhash Kumar Yadav [8], Khan. M and Shabbir. J [9], Jeelani. Iqbal and Maqbool. S [10], have used different population parameters as auxiliary variables to improve the precision and efficiency of variance estimators. Similarly Bhat et al. (2018) [11] have used linear combination of skewness and quartiles as auxiliary information to obtain the precision of estimators.

Let the finite population under survey be $U = \{U_1, U_2, ..., U_N\}$, consists of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i = 1,2,3........N$, giving a vector $Y = \{y_1, y_2, ..., y_N\}$. The goal is to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{I=1}^{N} y_i$ or its variance $S_Y^2 = \frac{1}{N-1}\sum_{I=1}^{N}(y_i - \bar{y})^2$ on the basis of random sample selected from a population U... In this paper, our aim is to estimate the precise and reliable estimates of finite population variance in the presence of outliers, as deciles are not sensitive to outliers.

2. MATERIALS AND METHODS

2.1 Notations

$N =$ Population size. $n =$ Sample size. $\gamma = \frac{1}{n}$ Y= study variable. X= Auxiliary variable. $\bar{X}$, $\bar{Y}$ = , Population means. $\bar{x}$, $\bar{y} =$ Sample means. S_Y^2 , $S_x^2 =$ population variances. s_y^2 , s_x^2 = sample variances. C_x , $C_y =$ Coefficient of variation. $\rho =$ Correlation coefficient. $\beta_{1(x)} =$ Skewness of the auxiliary variable. $\beta_{2(x)} =$ Kurtosis of the auxiliary variable. $\beta_{2(y)} =$ Kurtosis of the study variable. M_d= Median of the auxiliary variable. B(.)=Bias of the estimator. MSE(.)= Mean square error. $\hat{S}_R^2 =$ Ratio type variance estimator. $\hat{S}_{Kc1}^2$, $\hat{S}_{jG}^2$,= Existing modified ratio estimators. *QD*= quartile deviation, $Q_{M.A}$ = quartile mean average, Q_1= first quartile, Q_2= second quartile, Q_3= third quartile, Q_r= quartile range.

Let we discuss first the already existing estimators in the literature and then the proposed estimators where, we have used the Linear Combination of standard deviation and deciles as auxiliary information to improve the efficiency of proposed estimators.

2.2 Existing Estimators

2.2.1 Ratio type variance estimator proposed by Isaki [1]

$$\hat{S}_{Is}^2 = s_y^2 \left[\frac{S_x^2}{s_x^2}\right] \quad (1)$$

The expressions for bias and mean square error of the estimator up to first order of approximation is given by

$$\text{Bias } (\hat{S}_{IS}^2) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \quad (2)$$

$$\text{MSE}((\hat{S}^2_{IS}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1) \right] \tag{3}$$

2.2.2 Ratio type variance estimator proposed by Upadhyaya and Singh [2]

$$\hat{S}^2_{US} = s_y^2 \left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)} \right] \tag{4}$$

The expressions for bias and mean square error of the estimator up to first order of approximation is give by

$$\text{Bias}((\hat{S}^2_{US}) = \gamma S_y^2 A_{US} \left[A_{US}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1) \right] \tag{5}$$

$$\text{MSE}((\hat{S}^2_{US}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{US}^2 (\beta_{2(x)} - 1) - 2A_{US}(\lambda_{22} - 1) \right] \tag{6}$$

2.2.3 Ratio type variance estimator proposed by Kadilar and Cingi [3]

$$\hat{S}^2_{kc1} = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x} \right] \tag{7}$$

The expressions for bias and mean square error of the estimator up to first order of approximation is given by

$$\text{Bias}((\hat{S}^2_{kc1}) = \gamma S_y^2 A_1 \left[A_1\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1) \right] \tag{8}$$

$$\text{MSE}((\hat{S}^2_{kc1}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_1^2 (\beta_{2(x)} - 1) - 2A_1(\lambda_{22} - 1) \right] \tag{9}$$

2.2.4 Ratio type variance estimator proposed by j. Subramani and Kumarapandiyan [4]

$$\hat{S}^2_{jG} = s_y^2 \left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1} \right] \tag{10}$$

The expressions for bias and mean square error of the estimator up to first order of approximation is given by

$$\text{Bias}((\hat{S}^2_{jG}) = \gamma S_y^2 A_{jG} \left[A_{jG}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1) \right] \tag{11}$$

$$\text{MSE}((\hat{S}^2_{jG}) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{jG}^2 (\beta_{2(x)} - 1) - 2A_{jG}(\lambda_{22} - 1) \right] \tag{12}$$

3. PROPOSED ESTIMATOR

We have proposed a class of modified ratio type variance estimators to estimate the finite population variance in the presence of outliers which are as follows:-

$$\hat{S}^2_{MS1} = s_y^2\left[\frac{S_x^2 + (S_x D_1)}{s_x^2 + (S_x D_1)}\right] \qquad \hat{S}^2_{MS2} = s_y^2\left[\frac{S_x^2 + (S_x D_2)}{s_x^2 + (S_x D_2)}\right]$$

$$\hat{S}^2_{MS3} = s_y^2\left[\frac{S_x^2 + (S_x D_3)}{s_x^2 + (S_x D_3)}\right] \qquad \hat{S}^2_{MS4} = s_y^2\left[\frac{S_x^2 + (S_x D_4)}{s_x^2 + (S_x D_4)}\right] \quad \hat{S}^2_{MS5} = s_y^2\left[\frac{S_x^2 + (S_x D_5)}{s_x^2 + (S_x D_5)}\right]$$

$$\hat{S}^2_{MS6} = s_y^2\left[\frac{S_x^2 + (S_x D_6)}{s_x^2 + (S_x D_6)}\right] \quad \hat{S}^2_{MS7} = s_y^2\left[\frac{S_x^2 + (S_x D_7)}{s_x^2 + (S_x D_7)}\right] \quad \hat{S}^2_{MS8} = s_y^2\left[\frac{S_x^2 + (S_x D_8)}{s_x^2 + (S_x D_8)}\right]$$

$$\hat{S}^2_{MS9} = s_y^2\left[\frac{S_x^2 + (S_x D_9)}{s_x^2 + (S_x D_9)}\right] \quad \hat{S}^2_{MS10} = s_y^2\left[\frac{S_x^2 + (S_x D_{10})}{s_x^2 + (S_x D_{10})}\right]$$

Here, the expressions for bias and mean square error of proposed estimators have been derived up to first order of approximation which is as follows:

$$\hat{S}^2_{MSi};\ i = 1,2,....,10$$

$$\hat{S}^2_{MSi} = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right] \tag{13}$$

$$\Rightarrow \quad \hat{S}^2_{MSi} = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right] \tag{14}$$

$$\Rightarrow \quad \hat{S}^2_{MSi} = \frac{S_y^2(1+e_0)}{(1+A_{MSi}e_1)} \text{ where } A_{MSi} = \frac{S_x^2}{S_x^2 + \alpha a_i} \tag{15}$$

Where $a_i = (S_x D_i);\ i = 1,2,3,....,10$

$$\Rightarrow \quad \hat{S}^2_{MSi} = S_y^2(1+e_0)(1+A_{MSi}e_1)^{-1}$$

$$\Rightarrow \quad \hat{S}^2_{MSi} = S_y^2(1+e_0)(1 - A_{MSi}e_1 + A^2_{MSi}e_1^2 - A^3_{MSi}e_1^3 + ...) \tag{16}$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}^2_{MSi} = S_y^2 + S_y^2 e_0 - S_y^2 A_{MSi}e_1 - S_y^2 A_{MSi}e_0 e_1 + S_y^2 A^2_{MSi}e_1^2$$

$$\Rightarrow \quad \hat{S}^2_{MSi} - S_y^2 = S_y^2 e_0 - S_y^2 A_{MSi}e_1 - S_y^2 A_{MSi}e_0 e_1 + S_y^2 A^2_{MSi}e_1^2 \tag{17}$$

By taking expectation on both sides of (5), we get

$$E(\hat{S}^2_{MSi} - S_y^2) = S_y^2 E(e_0) - S_y^2 A_{MSi}E(e_1) - S_y^2 A_{MSi}E(e_0 e_1) + S_y^2 A^2_{MSi}E(e_1^2) \tag{18}$$

$$Bias(\hat{S}^2_{MSi}) = S_y^2 A^2_{MSi}E(e_1^2) - S_y^2 A_{MSi}E(e_0 e_1)$$

$$Bias(\hat{S}^2_{MSi}) = \gamma\, S_y^2 A_{MSi}[A_{MSi}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \tag{19}$$

Squaring both sides of (5) and (6), neglecting the terms more than 2[nd] order and taking expectation, we get

$$E(\hat{S}_{MSi}^2 - S_y^2)^2 = S_y^4 E(e_0^2) + S_y^4 A_{MSi}^2 E(e_1^2) - 2S_y^4 A_{MSi} E(e_0 e_1)$$

$$MSE(\hat{S}_{MSi}^2) = \gamma S_y^4 [(\beta_{2(y)} - 1) + A_{MSi}^2 (\beta_{2(x)} - 1) - 2A_{MSi} (\lambda_{22} - 1)]$$

4. RESULTS AND DATA ANALYSIS

4.1 Numerical Illustration

We use the data of Singh and Chaudhary (1986). We apply the proposed and existing estimators to this data set and the data statistics is given below:

Population Singh and Chaudhary [5]:

$$N = 34, n = 20, S_x = 15.0506, C_x = 0.7205, S_y = 73.314, \beta_{1x} = 0.8732, \beta_{2x} = 2.9123, \beta_{2y} = 13.3666,$$
$$\overline{X} = 20.8882, \overline{Y} = 85.6412, \lambda_{22} = 1.1525, D_1 = 7.03, D_2 = 7.68, D_3 = 10.82, D_4 = 12.94, D_5 = 15.0,$$
$$D_6 = 22.72, D_7 = 25.04, D_8 = 33.56, D_9 = 43.61, D_{10} = 56.4$$

Table 1. Bias and Mean square error of existing and proposed estimators

Estimators	Bias	Mean Square Error
Isaki [1]	194.37	8895573.118
Upadhyaya and Singh [2]	189.6191	8285277.5233
Kadilar&Cingi [3]	193.4567	8305040.9247
Subramani&Kumarapandiyan [4]	194.2999	8309384.7128
Proposed (MS1)	86.4361	7727064.6190
Proposed (MS2)	81.2421	7700413.4419
Proposed (MS3)	64.0955	7419391.5854
Proposed (MS4)	51.8828	7561175.8477
Proposed (MS5)	44.4265	7517823.2662
Proposed (MS6)	26.7472	7431828.8014
Proposed (MS7)	23.3830	7415897.3200
Proposed (MS8)	14.9929	7375446.7556
Proposed (MS9)	9.7355	7353059.7668
Proposed (MS10)	5.8059	7336239.9128

Table 2. Percent relative efficiency of existing estimators with proposed estimators

Existing estimators $\rightarrow$ *Proposed estimators* $\downarrow$	**Isaki [1]**	**Upadhyaya And Singh [2]**	**Kadilar and Cingi [3]**	**J. Subramani and Kumarapandiyan. G [4]**
P_1	115.1209	107.2247	107.4749	107.5356
P_2	115.5232	107.5952	107.8518	107.9078
P_3	119.8921	111.6705	111.9369	111.9950
P_4	117.6430	109.5765	109.8379	109.8949
P_5	118.3260	110.2084	110.4713	110.5286
P_6	119.6975	111,4836	111.7496	111.8080
P_7	119.9587	111.7231	111.9896	112.0482
P_8	120.6176	112.3359	112.6039	112.6027
P_9	120.9780	112.6779	112.9467	113.0058
P_{10}	121.2553	112.9362	113.2056	113.2648

5. CONCLUSION

The modified ratio type variance estimators in this paper proposed have been found more efficient than the existing estimators by using the known parameters of the auxiliary variable. Also from Table 1 and Table 2 Bias, mean square error and percent relative efficiency of existing and proposed estimators clearly shows that the proposed modified ratio type variance estimators are more efficient as compared to mentioned existing estimators. Thus, it is recommended that our proposed estimators may be preferred over the existing estimators for use in practical applications.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Isaki CT. Variance estimation using auxiliary information. Journal of the American Statistical Association. 1983;78:117-123.
2. Upadhyaya LN, Singh HP. Use of auxiliary variable in the estimation of population variance, Mathematical forum. 1999;1936:4:33-36.
3. Kadilar C, Cingi H. Improvement in Variance estimation using auxiliary information. Hacettepe Journal of Mathematics and Statistics. 2006a;35(1):117-115.
4. Subramani J, Kumarapandiyan G. Generalized modified ratio type estimator for estimation of population variance. Sri-Lankan Journal of Applied Statistics. 2015;16-1:69-90.
5. Singh D, Chaudhary FS. Theory and analysis of sample survey designs. New age publishers; 1986.
6. Murthy MN. Sampling theory. Theory and Methods, Statistical publishing Society. Calcutta;1967.
7. Arcos A, Rueda M, Martinez MD, Gonzalez S and Roman Y. Incorporatin the auxiliary information available in variance estimation. Applied Mathematical and Computation. 2005.160: 387-399.
8. Subash Kumar Yadav, Sheela Misra, Mishra SS. American Journal of operational Research. 2016;6(1):9-15.
9. Khan M, Shabbir JA. A ratio type Estimator for the Estimation of population variance using as an auxiliary variable. Journal of Statistics Applications & Probability. 2013;2:319-325.
10. Jeelani Iqbal, Maqbool S. Modified ratio estimators of population mean using linear combination of coefficient of skewness and quartile deviation. The South Pacific Journal of Natural and Applied Sciences. 2013;31:39-44.
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