Asian Research Journal of Mathematics

11(3): 1-12, 2018; Article no.ARJOM.44676

ISSN: 2456-477X

An Order Six Stormer-cowell-type Method for Solving Directly Higher Order Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author SJK designed the study and wrote the first draft of the manuscript. Authors OSI and FOO managed the analyses and the literature searches while author EOO managed the numerical implementation of the study. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2018/44676
Editor(s):
(1) Dr. Radoslaw Jedynak, Department of Computer Science and Mathematics, Kazimierz Pulaski University of Technology and Humanities, Poland.
Reviewers:
(1) Zeraoulia Elhadj, University of Tébessa, Algeria.
(2) Samson Omagwu, Kaduna Polytechnic, Nigeria.
(3) Aliyu Bhar Kisabo, Centre for Space Transport and Propulsion, Nigeria.
Complete Peer review History: http://www.sciencedomain.org/review-history/27323

Original Research Article

Received: 06 September 2018
Accepted: 11 November 2018
Published: 20 November 2018

Abstract

This paper considers the development of an efficient Stormer-Cowell-Typed method for the direct solution of second order ordinary differential equations using the method of interpolation of the combination of Cheybeshev and Legendre polynomials approximate solution and collocation of the differential system to develop our scheme. The method derived was tested and confirmed to be consistent, stable within the region of absolute stability and zero-stable. The method was tested on some numerical examples and found to give a better approximation.

Keywords: General second order; interpolation; Chebyshev; Legendre; collocation; order; zero stability; consistent.

1 Introduction

Several fields of applications, notably in science and engineering yield initial value problems of second order differential equations are of the form

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$$y'' = f(x, y, y'); \;\; y(a) = y_0; \;\; y'(a) = y_1, \;\; f \in c^2(a \;,\; b) \tag{1}$$

Countless of such problem may not easily be solved analytically, thus numerical schemes are developed to solve (1). These equations are usually reduced to systems of first-order ordinary equations and numerical scheme of the first order differential equation are employed to solve them. Linear multistep methods are an influential numerical method for solving differential equation than the explicit method. Kayode and Adeyeye [1] reported that literature revealed that some researchers have attempted the direct solution of (1) using linear multistep method (Lambert [2], Brown [3], Awoyemi [4], Adesanya et al. [5], Kayode [6], Alabi et al. [7], Kayode and Obarhua [8]) with the various order of accuracies. It was also reported in Kayode and Adeyeye [1], that lower order method was developed by Kayode [9], Yahaya and Badmus [10], Majid et al. [11], Ehigie et al. [12], Kayode and Adeyeye [13], to solve (1). Kayode and Adeyeye [1] proposed two-step two-point hybrid methods for general second order ordinary differential equations which chebyshev polynomial of the first kind was used as basis function and the method was of order six. The method was used to solve the same problem treated by the method of Awoyemi [14], Yahaya and Badmus [10] and Ehigie et al. [12] the error compared favourably well to that of Awoyemi [14], Yahaya and Badmus [10] and Ehigie et al. [12]. Recently, Omole and Ogunware [15], worked on 3-

Point Single Hybrid Point Block Method (3PSHBM) for direct solution of General second order initial value problem of ordinary differential equations. The method was found to be zero stable, consistency and efficient for solving initial value problems accurately.

This work made use of Chebsyshev and Legendre polynomials as basis function in generating the interpolation and collocation equations for the development of a continuous Linear multistep method of Stormer-Cowell type for the direct solution of (1) which is of the higher order.

2 The Derivation of the Method

In this section, we apply the interpolation and collocation technique and we chose our interpolation (i) and our collocation points (c) at grid points. We considered the combination of Chebyshev and Legendre Polynomials in the form

$$y(x) = \sum_{j=0}^{c+i-1} a_j \left\{ T_j(x) + P_j(x) \right\} \tag{2}$$

where $T_j(x)$ is the Chebyshev polynomial of the first kind and $P_j(x)$ is the legendre polynomial. Equation (2) is the basis function with a single variable x, where $x \in (a,b)$, a' s are real unknown parameter to be determined, c and i are the number of collocation and interpolation points respectively.

The second derivative of (2) is

$$y''(x) = \sum_{j=0}^{c+i-1} a_j \left\{ T_j''(x) + P_j''(x) \right\} \tag{3}$$

Combining (3) in (1) to have

$$\sum_{j=0}^{c+i-1} a_j T_j''(x) + \sum_{j=0}^{c+i-1} a_j P_j''(x) = f(x, y, y') \tag{4}$$

Collocated (4) at $x_{n+i}, i = 0(1)5$ and interpolated (2) at $x_{n+i}, i = 3,4$ give rise to the following set of equations

$$f_n = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_n + \frac{1188}{8}a_4x^2{}_n - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_n - \frac{1380}{8}a_5x_n + \frac{22290}{16}a_6x^4{}_n \\ -\frac{12996}{16}a_6x^2{}_n - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_n - \frac{49700}{16}a_7x^4{}_n - \frac{7266}{16}a_7x_n \end{pmatrix}$$

$$f_{n+1} = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_{n+1} + \frac{1188}{8}a_4x^2{}_{n+1} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_{n+1} - \frac{1380}{8}a_5x_{n+1} + \frac{22290}{16}a_6x^4{}_{n+1} \\ -\frac{12996}{16}a_6x^2{}_{n+1} - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_{n+1} - \frac{49700}{16}a_7x^4{}_{n+1} - \frac{7266}{16}a_7x_{n+1} \end{pmatrix}$$

$$f_{n+2} = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_{n+2} + \frac{1188}{8}a_4x^2{}_{n+2} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_{n+2} - \frac{1380}{8}a_5x_{n+2} + \frac{22290}{16}a_6x^4{}_{n+2} \\ -\frac{12996}{16}a_6x^2{}_{n+2} - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_{n+2} - \frac{49700}{16}a_7x^4{}_{n+2} - \frac{7266}{16}a_7x_{n+2} \end{pmatrix}$$

$$f_{n+3} = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_{n+3} + \frac{1188}{8}a_4x^2{}_{n+3} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_{n+3} - \frac{1380}{8}a_5x_{n+3} + \frac{22290}{16}a_6x^4{}_{n+3} \\ -\frac{12996}{16}a_6x^2{}_{n+3} - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_{n+3} - \frac{49700}{16}a_7x^4{}_{n+3} - \frac{7266}{16}a_7x_{n+3} \end{pmatrix}$$

$$f_{n+4} = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_{n+4} + \frac{1188}{8}a_4x^2{}_{n+4} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_{n+4} - \frac{1380}{8}a_5x_{n+4} + \frac{22290}{16}a_6x^4{}_{n+4} \\ -\frac{12996}{16}a_6x^2{}_{n+4} - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_{n+4} - \frac{49700}{16}a_7x^4{}_{n+4} - \frac{7266}{16}a_7x_{n+4} \end{pmatrix}$$

$$f_{n+5} = \begin{pmatrix} 7a_2 + \frac{78}{2}a_3x_{n+5} + \frac{1188}{8}a_4x^2{}_{n+5} - \frac{188}{4}a_4 + \frac{3820}{8}a_5x^3{}_{n+5} - \frac{1380}{8}a_5x_{n+5} \\ +\frac{22290}{16}a_6x^4{}_{n+5} - \frac{12996}{16}a_6x^2{}_{n+5} - \frac{786}{16}a_6 + \frac{61026}{16}a_7x^5{}_{n+5} - \frac{49700}{16}a_7x^4{}_{n+5} - \frac{7266}{16}a_7x_{n+5} \end{pmatrix}$$

$$y_{n+3} = \begin{pmatrix} 2a_0 + 2a_1x_{n+3} + \frac{7}{2}a_2x^2{}_{n+3} - \frac{3}{2}a_2 + \frac{13}{2}a_3x^3{}_{n+3} - \frac{9}{2}a_3x_{n+3} + \\ \frac{99}{8}a_4x^4{}_{n+3} - \frac{94}{8}a_4x^2{}_{n+3} + \frac{11}{8}a_4 + \frac{191}{8}a_5x^5{}_{n+3} - \frac{230}{8}a_5x^3{}_{n+3} \\ +\frac{55}{8}a_5x_{n+3} + \frac{743}{16}a_6x^6{}_{n+3} - \frac{1083}{16}a_6x^4{}_{n+3} - \frac{393}{16}x^2{}_{n+3}a_6 - \frac{21}{16}a_6 \\ +\frac{1457}{16}a_7x^7{}_{n+3} - \frac{2485}{16}a_7x^5{}_{n+3} + \frac{1211}{16}a_7x^3{}_{n+3} - \frac{147}{16}a_7x_{n+3} \end{pmatrix} \quad (5)$$

$$y_{n+4} = \begin{pmatrix} 2a_0 + 2a_1 x_{n+4} + \frac{7}{2} x^2{}_{n+4}\, a_2 - \frac{3}{2} a_2 + \frac{13}{2} a_3 x^3{}_{n+4} - \frac{9}{2} a_3 x_{n+4} + \\ \frac{99}{8} a_4 x^4{}_{n+4} - \frac{94}{8} a_4 x^2{}_{n+4} + \frac{11}{8} a_4 + \frac{191}{8} a_5 x^5{}_{n+4} - \frac{230}{8} a_5 x^3{}_{n+4} \\ + \frac{55}{8} a_5 x_{n+4} + \frac{743}{16} a_6 x^6{}_{n+4} - \frac{1083}{16} a_6 x^4{}_{n+4} - \frac{393}{16} a_6 x^2{}_{n+4} - \frac{21}{16} a_6 \\ + \frac{1457}{16} a_7 x^7{}_{n+4} - \frac{2485}{16} a_7 x^5{}_{n+4} + \frac{1211}{16} a_7 x^3{}_{n+4} - \frac{147}{16} a_7 x_{n+4} \end{pmatrix}$$

Equation (5) is solved by Gaussian elimination method to attain the value of the unknown parameters a_j , j= 0(1)7as follows

$$a_0 = \frac{1}{2h} \begin{pmatrix} y_{n+3} - 2a_1 x_{n+3} + \frac{7}{2} a_2 x^2{}_{n+3} - \frac{3}{2} a_2 + \frac{13}{2} a_3 x^3{}_{n+3} - \frac{9}{2} a_3 x_{n+3} \\ + \frac{99}{8} a_4 x^4{}_{n+3} - \frac{94}{8} a_4 x^2{}_{n+3} + \frac{11}{8} a_4 + \frac{191}{8} a_5 x^5{}_{n+3} - \frac{230}{8} a_5 x^3{}_{n+3} \\ + \frac{55}{8} a_5 x_{n+3} + \frac{743}{16} a_6 x^6{}_{n+3} - \frac{1083}{16} a_6 x^4{}_{n+3} - \frac{393}{16} a_6 x^2{}_{n+3} - \frac{21}{16} a_6 \\ + \frac{1457}{16} a_7 x^7{}_{n+3} - \frac{2485}{16} a_7 x^5{}_{n+3} + \frac{1211}{16} a_7 x^3{}_{n+3} - \frac{147}{16} a_7 x_{n+3} \end{pmatrix}$$

$$a_1 = \frac{1}{2h} \begin{pmatrix} y_{n+4} - y_{n+3} - \frac{14}{2} a_2 x_n h - \frac{49}{2} a_2 h^2 - \frac{568}{2} a_3 x_n{}^3 h - \frac{364}{2} a_3 x^2{}_n h^2 - \frac{286}{2} a_3 h^3 + \frac{9}{2} a_3 x_3 h \\ - \frac{529}{8} a_4 x_n{}^{3`} h - \frac{3250}{8} a_4 x_n{}^2 h^2 + \frac{9524}{8} a_4 x_n\, h^3 - \frac{8354}{8} a_4 h^4 + \frac{188}{8} a_4 x_n h - \frac{520}{8} a_4 h^2 \\ - \frac{1055}{8} a_5 x_n{}^4 h - \frac{12550}{8} a_5 x_n{}^3 h^2 - \frac{5281}{8} a_5 x_n{}^2 h^3 - \frac{9525}{8} a_5 x_n h^4 - \frac{6320}{8} a_5 h^5 \\ + \frac{890}{8} a_5 x_5{}^2 h + \frac{5420}{8} a_5 x_n h^2 + \frac{5370}{8} a_5 h^3 - \frac{55}{8} a_5 h + \frac{6558}{16} a_6 x_n{}^5 h + \frac{85725}{16} a_6 x_n{}^4 h^2 + \\ \frac{124860}{16} a_6 x^3{}_n h^3 - \frac{923525}{16} a_6 x^2{}_n h^4 - \frac{1020355}{16} a_6 x_n h^5 + \frac{584395}{16} a_6 h^6 + \frac{29332}{16} a_6 x^3{}_n h \\ + \frac{624940}{16} a_6 x^2{}_n h^2 - \frac{90395}{16} a_6 h^3 - \frac{988}{16} a_6 x_n h - \frac{2866}{16} a_6 h^2 - \frac{40684}{16} a_7 x^6{}_n h \\ - \frac{488208}{16} a_7 x^5{}_n h^2 - \frac{1952832}{16} a_7 x^4{}_n h^3 + \frac{3835694}{16} a_7 x^3{}_n h^4 - \frac{2463816}{16} a_7 x^2{}_n h^5 \\ - \frac{458351}{16} a_7 x_n h^6 - \frac{7531}{16} a_7 h^7 - \frac{436151}{16} a_7 x^4{}_n - \frac{83451}{16} a_7 x^3{}_n h^2 - \frac{163531}{16} a_7 x_n h^3 \\ - \frac{25341}{16} a_7 x_n h^4 - \frac{54321}{16} a_7 h^5 + \frac{6354}{16} a_7 x^2{}_n h + \frac{3321}{16} a_7 x_n h^2 + \frac{12622}{16} a_7 h^3 - \frac{147}{16} a_7 h \end{pmatrix}$$

$$a_2 = \frac{1}{7h} \begin{pmatrix} f_n - \frac{78}{2} a_3 x_n - \frac{1188}{2} x_n{}^2 a_4 + \frac{188}{8} a_4 - \frac{3820}{8} a_5 x_n{}^3 + \frac{1380}{8} x_n\, a_5 - \frac{22290}{16} a_6 x_n{}^4 \\ - \frac{12996}{16} a_6 x_n{}^2 + \frac{786}{16} a_6 - \frac{61026}{16} a_7 x_n{}^5 - \frac{49700}{16} a_7 x_n{}^3 - \frac{7266}{16} a_7 x_n \end{pmatrix}$$

$$a_3 = \frac{2}{78h}\left(\begin{array}{l} f_{n+1} - f_n - \frac{2376}{8}a_4x_nh - \frac{1188}{8}a_4h^2 - \frac{11460}{8}a_5{x_n}^2h - \frac{11460}{8}a_5x_nh^2 - \frac{3820}{8}a_5h^3 \\ + \frac{1380}{8}a_5h - \frac{89160}{16}a_6{x_n}^3h - \frac{133740}{16}a_6{x_n}^2h^2 - \frac{89160}{16}a_6x_nh^3 - \frac{22290}{16}a_6h^4 \\ + \frac{25992}{16}a_6x_nh + \frac{12996}{8}a_6h^2 - \frac{305130}{16}a_7{x_n}^4h - \frac{610260}{16}a_7{x_n}^3h^2 - \frac{610260}{16}a_7x^2{}_nh^3 \\ - \frac{305130}{16}a_7x_nh^4 - \frac{61026}{16}a_7h^5 - \frac{198800}{16}a_7{x_n}^3h - \frac{298200}{16}a_7x^2{}_nh^2 - \frac{19880}{16}a_7x_nh^3 \\ - \frac{49700}{16}a_7h^4 - \frac{7266}{16}a_7h \end{array}\right)$$

$$a_4 = \frac{8}{2376h^2}\left(\begin{array}{l} f_{n+2} - 2f_{n+1} + f_n - \frac{22920}{8}a_5x_nh^2 - \frac{22920}{8}a_5h^3 - \frac{267480}{16}a_6x^2{}_nh^2 \\ - \frac{534960}{16}a_6x_nh^3 - \frac{312060}{16}a_6h^4 - \frac{233463}{16}a_7{x_n}^3h^4 - \frac{345212}{16}a_7h^5 \end{array}\right)$$

$$a_5 = \frac{8}{32720h^3}\left(f_{n+3} - 3f_{n+2} + 3f_{n+1} - f_n - \frac{534960}{16}a_6x_nh^3 - \frac{802440}{16}a_6h^4 - \frac{145635}{16}a_7h^5\right)$$

$$a_6 = \frac{8}{534960h^4}\left(f_{n+4} - 4f_{n+3} + 4f_{n+2} - 4f_{n+1} + f_n - -\frac{258346}{16}a_7x_nh^4 - \frac{184361}{16}a_7h^5\right)$$

$$a_7 = \frac{8}{1635480h^2}\left(f_{n+5} - 5f_{n+4} + 5f_{n+3} + 5f_{n+2} - 5f_{n+1} + f_n\right) \tag{6}$$

The a_j's are replaced back into (2) and simplifying to give a continuous method of the type

$$y_{n+k}(x) = \sum_{j=k-2}^{k-1}\alpha_j y_{n+j}(x) + h^2 \sum_{j=0}^{k}\beta_j(x) f_{n+j} \tag{7}$$

Applying the transformation in Kayode & Obarhua (2013), $t = \frac{x - x_{n+k-1}}{h}$ and $dt = \frac{1}{h}dh$, the coefficients are given as follows

$$\alpha_3 = t$$

$$\alpha_4 = -(t+1)$$

$$\beta_0 = h^2\left(-\frac{220}{360} - \frac{150}{360}t - \frac{990}{360}t^2 + \frac{160}{360}t^3 - \frac{160}{360}t^4 + \frac{290}{360}t^5 - \frac{30}{360}t^6 + \frac{720}{360}t^7\right)$$

$$\beta_1 = h^2\left(\frac{2806}{87654} - \frac{234}{24}t - \frac{1231}{4353}t^2 + \frac{403}{240}t^3 - \frac{263}{1240}t^4 - \frac{1380}{240}t^5 + \frac{133}{120}t^6 - \frac{108}{1220}t^7\right)$$

$$\beta_2 = h^2\left(-\frac{825}{343413}+\frac{360}{960}t \; -\frac{883}{11580}t^2+\frac{431}{240}t^3+\frac{18}{960}t^4-\frac{183}{120}t^5+\frac{1380}{240}t^6+\frac{624}{11580}t^7\right)$$

$$\beta_3 = h^2\left(\frac{1302}{5760}-\frac{831}{24}t+\frac{434}{120}t^2+\frac{1331}{413464}t^3-\frac{133}{960}t^4-\frac{1342}{1240}t^5-\frac{218}{120}t^6+\frac{321}{124}t^7\right)$$

$$\beta_4 = h^2\left(\frac{1146}{435842}-\frac{1435}{960}t+\frac{8316}{1435}t^2-\frac{438}{240}t^3+\frac{1310}{120}t^4+\frac{131}{24}t^5-\frac{808}{4325}t^6+\frac{188}{14354}t^7\right) \quad (8)$$

$$\beta_5 = h^2\left(\frac{163}{264556}+\frac{4330}{1335}t-\frac{334}{960}t^2+\frac{335}{240}t^3-\frac{8341}{16455}t^4+\frac{926}{120}t^5-\frac{342}{1335}t^6+\frac{163}{960}t^7\right)$$

The first derivative of (8) gives

$$\alpha_3' = \frac{1}{h}$$

$$\alpha_4' = -\frac{1}{h}$$

$$\beta_0' = h\left(\frac{150}{360}-\frac{1980}{360}t \; +\frac{480}{36}t^2-\frac{640}{360}t^3+\frac{1450}{360}t^4-\frac{180}{360}t^5+\frac{5040}{360}t^6\right)$$

$$\beta_1' = h\left(-\frac{234}{360}-\frac{2462}{4353}t \; +\frac{1209}{120}t^2-\frac{1052}{1240}t^3+\frac{1605}{720}t^4+\frac{798}{120}t^5-\frac{756}{1220}t^6\right)$$

$$\beta_2' = h\left(\frac{360}{960}-\frac{1676}{11580}t \; +\frac{1293}{240}t^2+\frac{72}{960}t^3-\frac{915}{120}t^4+\frac{8280}{240}t^5+\frac{4368}{11580}t^6\right)$$

$$\beta_3' = h\left(-\frac{831}{24}+\frac{868}{120}t \; +\frac{3993}{413464}t^2-\frac{532}{980}t^3+\frac{6710}{1240}t^4-\frac{1308}{120}t^5-\frac{2247}{124}t^6\right)$$

$$\beta_4' = h\left(-\frac{435}{960}+\frac{16632}{14354}t \; +\frac{1314}{240}t^2+\frac{5240}{120}t^3+\frac{655}{124}t^4-\frac{4848}{4325}t^5+\frac{1316}{1435}t^6\right)$$

$$\beta_5' = h\left(\frac{4330}{1335}-\frac{668}{960}t \; +\frac{1005}{240}t^2-\frac{33364}{16455}t^3+\frac{4630}{120}t^4-\frac{2052}{1335}t^5+\frac{1141}{960}t^6\right)$$

Evaluating (9) and (10) at $t=1$ which implies that $x = x_{n+5}$ gives discrete scheme

$$y_{n+5} = 2y_{n+4} - y_{n+3} + \frac{h^2}{240}\left(18f_{n+5} + 209f_{n+4} + 4f_{n+3} + 14f_{n+2} - 6f_{n+1} + f_n\right) \tag{10}$$

The first derivative is

$$y'_{n+5} = \frac{1}{h}\left(-y_3 + y_4\right) + \frac{h}{10080}\begin{pmatrix}3218f_{n+5} + 13093f_{n+4} - 2876f_{n+3} + 2470f_{n+2} \\ -934f_{n+1} + 149f_n\end{pmatrix} \tag{11}$$

The Predictor

$$y_{n+5} = 2y_{n+4} - y_{n+3} + \frac{h^2}{240}\left(299f_{n+4} - 176f_{n+3} + 194f_{n+1} - 96f_{n+1} + 19f_n\right) \tag{12}$$

with its first derivative as

$$y'_{n+5} = \frac{1}{h}\left(-y_3 + y_4\right) + \frac{h}{1440}\left(4169f_{n+4} - 5008f_{n+3} + 4950f_{n+2} - 2432f_{n+1} + 481f_n\right) \tag{13}$$

Other explicit systems were generated to evaluate the remaining values using Taylor series.

$$y_{n+j} = y_n + (jh)y_n' + \frac{(jh)^2}{2!}f_n + \frac{(jh)^3}{3!}\left\{\frac{\partial f_n}{\partial x_n} + y'_n\frac{\partial f_n}{\partial y_n} + f_n\frac{\partial f_n}{\partial y'_n}\right\} + 0\left(h^4\right)$$

and

$$y'_{n+j} = y'_n + (jh)f_n + \frac{(jh)^2}{2!}\left\{\frac{\partial f_n}{\partial x_n} + y'_n\frac{\partial f_n}{\partial y_n} + f_n\frac{\partial f_n}{\partial y'_n}\right\} + 0\left(h^3\right)$$

3 Analysis of the Basic properties of the Method

3.1 Order and error constant of the method

We embrace the method proposed by Lambert [2], in finding the order, with the operator:

We associate the linear operator L with the continuous multistep method (7) and defined as

$$L\{y(x)\,;\,h\} = \sum_{j=0}^{k}\left\{\alpha_j y(x+jh) - h\beta_j y''(x+jh)\right\}$$

where α_j and β_j are both non-zero and $y(x)$ is an arbitrary function, continuously differentiable on the interval [a, b].If we assume that $y(x)$ has many higher derivatives as we required, then on Taylor series expansion about x, we obtain

$$L[y(x),h]=C_0y(x)+C_1hy^1(x)+C_2h^2y^{(2)}+\ldots C_Ph^py^{(p)}(x) \tag{14}$$

where the C_p ir constants.

Therefore we say that the method has order P if,

$$C_0=C_1=C_2=\ldots C_P=C_{P+1}=0, C_{P+2}\neq 0$$

Then, C_{P+2} is the error constant, and it implies that the principal local truncation error is given by $C_{P+2}h^{p+2}y^{(p+2)}(x_n)$

For our method

$$C_0=C_1=C_2=C_3=C_4=C_5=C_6=C_7=0, C_8=-\frac{221}{60480}.$$

Therefore the derived scheme is of order 6

4 The Consistency of the Method

For our method to be consistent, the following conditions must be satisfied

(i) the order $\rho \geq 1$

(ii) $\sum_{j=0}^{k} \alpha_j = 0$

(iii) $\rho(1)=\rho'(1)=0$

(iv) $\rho''(1)=2\ !\sigma(1)$

Condition (i) is satisfied since the scheme is of order 6

Condition (ii) is satisfied since $\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5=0;\ 0+0+0+1-2+1=0$

Condition (iii) is satisfied since $\rho(r)=r^5-2r^4+r^3 and\ \rho'(r)=5r^4-8r^3+3r^2$

when r =1; $\rho(r)=\rho'(r)=0$

Condition (iv) is satisfied since

$$\rho^{''}(r) = 20r^3 - 24r^2 + 6r \ and \ \sigma(r) = \frac{1}{240}\left(18r^5 + 209r^4 + 4r^3 + 14r^2 - 6r + 1\right)$$

when r =1; $\sigma(1) = 2! \times \left(\frac{18}{240} + \frac{209}{240} + \frac{4}{240} + \frac{14}{240} - \frac{6}{240} + \frac{1}{240}\right) = 2 \ \times \ \frac{240}{240} = 2 \times 1 = 2$

Therefore $\rho^{''}(r) = 2 \ ! \sigma(r) = 2$

Hence the four conditions are satisfied, the method is consistent

5 Zero Stability

Definition: A linear multistep method is said to be zero-stable, if no root of the first characteristics polynomial $\rho(r)$ has a modulus greater than one and if every root of modulus one has multiplicity not greater than two. The scheme is zero stable when no root of the first characteristics polynomial has a modulus greater than one that is $|r| \ \leq \ 1$.

A method is zero stable if $\rho(x) = \sum_{j=0}^{k} \alpha_j \ = 0$, where α_j are the coefficients of $\sum_{j=0}^{k} \alpha_j y_{n+j}$

$$\sum_{j=0}^{k} \alpha_j = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0 + 0 + 0 + 1 - 2 + 1 = 0$$

$$\rho(r) = r^5 - 2r^4 + r^3 = 0$$
$$(r^2 - 2r + 1) = 0$$
$$(r - r)(r - 1) = 0$$
$$r = 1 \ \ twice$$

Thus, the method is zero stable.

6 The Region of Absolute Stability

The Equation (11) is said to be stable if for a given h all the roots z_n of the characteristics

Polynomial $\sigma(z, \hbar) = \rho(z) + h\sigma(z) = 0$ satisfies $|z_s| \ \prec 1, \ s = 1,2...,n \ where \ \hbar = \lambda h$

We adopted the boundary locus method to determine the stability interval. Substituting the test equation $y' = -\lambda y$ into equation (11) gives $\hbar(r, h) = \frac{\rho(r)}{\sigma(r)}$. Writing $r = e^{i\theta}$. After simplification, the stability interval gives $[0, \ -4.615,]$ after evaluating $\hbar(r, \ h)$ at interval 30^0. Hence the method is p-stable in nature.

7 Implementation of the Method

Problem 1

$$y'' = y' \quad y(0) = 0, \quad y'(0) = -1 \qquad h = 0.1$$

$$\textit{Analytical solution}: y(x) = 1 - \exp(x)$$

Problem 2

$$y'' - x(y')^2 = 0 \quad y(0) = 1, \quad y'(0) = \frac{1}{2} \qquad h = 0.003125$$

$$\textit{Analytical solution}: y(x) = 1 + \frac{1}{2} In\left(\frac{2+x}{2-x}\right)$$

7.1 Numerical solutions to problem 1-2 as shown in Table 1-2

The computational errors of our method tested on problems 1-2 compared to other researchers. Problem 1 was compared with Kayode and Adeyeye [1]. Problem 2 was compared with Awoyemi [14] and Kayode and Adeyeye [13].

8 Results and Discussion

Table 1. Table for problem 1

X	Error in Kayode and Adeyeye [1]	Error in New method
0.2	8.17176E-07	-
0.3	3.10356E-06	-
0.4	6.56957E-06	-
0.5	1.14380E-05	5.709190E-9
0.6	1.79656E-05	2.084567E-9
0.7	2.64474E-05	3.066035E-9
0.8	3.72222E-05	5.020548E-9
0.9	5.06786E-05	5.320548E-9
1.0	6.72615E-05	8.021400E-9

Table 2. Table for problem 2

X	Error in Awoyemi [14]	Error in Kayode and Adeyeye [13]	Error in New method
0.0063	0.26075253e-09	4.831380e-11	9.325873e-15
0.0094	0.19816704e-08	3.382836e-09	1.865175e-14
0.0125	0.65074122e-08	1.580320e-08	2.797762e-14
0.0156	0.15592381e-08	4.333951e-08	3.730349e-14
0.0188	0.31504477e-08	9.391426e-08	4.662937e-14

9 Conclusion

In this work, we have derived, analysed and implemented an efficient stormer-cowell-type method for the solution of general second order ordinary differential equations by adopting a combination of Chebyshev and Legendre polynomials as the basis function. Collocation and interpolation methodology is adopted for the derivation of the method. In Table 1, our method performs better than the method of Kayode and Adeyeye [1], likewise Table 2; showed better accuracy than Awoyemi [14] and Kayode & Adeyeye [13]. Thus, the method developed in this paper is efficient and compared favourably well. The stability region shows that the method is P-stable.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Kayode SJ, Adeyeye O. Two-step two-point hybrid methods for general second order differential equations. Africa Journal of Computer and Mathematics Science Research; 2013.

[2] Lambert JD. Computational methods in ordinary differential equations. John Wiley & Sons Inc, New York; 1973.

[3] Brown RL. Some characteristics of implicit multistep multi derivative integration formula. SIAM J. Num. Anal. 1977;14:982-993.

[4] Awoyemi DO. A p-stable linear multistep method for solving third order ordinary differential equations. Inter. J. Comp. Math. 2003;80(8):85-991.

[5] Adesanya AO, Anake TA, Udoh MO. Improved continuous method for direct solution of general second order ordinary differential equation. J. Nig. Assoc. Math. Phys. 2008;13:59-62.

[6] Kayode SJ. A zero-stable optimal order method for direct solution of second order differential equations. J. Math. Stat. 2010;6(3):367-371.

[7] Alabi MO, Oladipo AT, Adesanya AO. Initial value solvers for second order ordinary differential equations using Chebyshev polynomial as basic functions. J. Mod. Math. Stat. 2008;2(1):18-27.

[8] Kayode SJ, Obarhua FO. 3 step y- function hybrid methods for direct numerical integration of second order IVPs in ODEs. Theoretical Mathematics and Application. 2013;5(1):39-51.

[9] Kayode SJ. A class of one-point zero-stable continuous hybrid methods for direct solution of second-order differential equations. Afric. J. Math. Comp. Sci. Res. 2011;4(3):93-99.

[10] Yahaya YA, Badmus AM. A class of collocation methods for general second order ordinary differential equations. Afric. J. Math.Comp. Sci. Res. 2009;2(4):69-72.

[11] Majid ZA, Azmi NA, Suleiman M. Solving second order ordinary differential equations using two point four step direct implicit block method. Euro. J. Sci. Res. 2009;31(1):29-36.

[12] Ehigie JO, Okunuga, SA, Sofoluwe AB, Akanbi MA. On generalized 2-step continuous linear multistep method of hybrid type for the integration of second order ordinary differential equation. Arch. Appl. Sci. Res. 2010;2(6):362-372.

[13] Kayode SJ, Adeyeye O. A 3-step hybrid method for direct solution of second order initial value problems. Aust. J. Basic Appl. Sci. 2011;5(12):2121-2126.

[14] Awoyemi DO. A p-stable linear multistep method for solving third order ordinary differential equations. Inter. J. Comp. Math. 2001;80(8):85-991.

[15] Omole EO, Ogunware BG. 3-point single hybrid point block method (3PSHBM) for direct solution of general second order initial value problem of ordinary differential equations. Journal of Scientific Research & Report. 2018;20(3):1-11.

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