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Modelling Volatility of Naira/US Dollar Exchange Rate Dynamics Using Conditional Heteroskedasticity Models with Non-Gaussian Errors

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Authors' contributions

This work was carried out in collaboration between both authors. Author DAK designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author PTA managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

This study searches for optimal symmetric and asymmetric Conditional Heteroskedasticity (ARCH/GARCH) models that best fit and model volatility between Nigeria Naira and United States Dollar exchange rate dynamics in Nigeria using non-Gaussian errors. The study utilizes daily closing Naira/US Dollar exchange rate data from 12/11/2001 to 12/01/2017 making a total of 3665 observations. Symmetric ARCH and GARCH, as well as asymmetric EGARCH and TGARCH specifications were used to model the log return series in the presence of student-t innovations and Generalized Error distribution. Results show that symmetric ARCH (3) and basic GARCH (1,1) with student-t innovations as well as asymmetric EGARCH (1,1) with GED distribution and TGARCH (1,1) with student-t innovation were the best fitting models for the Naira/US Dollar exchange rate log return series. All the estimated models were found to be unstable and non-stationary indicating over persistence of volatility shock in the conditional variance. The asymmetric EGARCH (1,1) and TGARCH (1,1) models show supportive evidence for the existence of asymmetry and leverage effects suggesting that negative shocks

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produce more volatility in Nigerian foreign exchange market than positive shocks of the same magnitude. The study provides policy recommendations for traders and investors in Nigerian exchange market.

Keywords: Exchange rate; GARCH models; Naira; Non-Gaussian errors; US dollar; volatility; Nigeria.

1 Introduction

The Nigerian economy is very sensitive to fluctuations in the Naira/US Dollar exchange rate given the fact that Nigeria depends significantly on imports and uses US dollars in the transaction; there is an increasing amount of foreign investment in the country and that important reserves are held in foreign exchange, especially US dollars. Moreover, banks as well as other financial institutions usually invest in foreign exchange instruments. The benefits of exchange rate stability to sustainable economic development in Nigeria and any other country cannot be overemphasized. The central bank of Nigeria (CBN) and other monetary agencies, therefore, try to control and avoid wide divergence between the official exchange rate and parallel exchange rates. However, the Nigerian Naira continues to fluctuate widely against the US dollar in spite of these policy efforts by CBN and other Nigeria monetary authorities to maintain stable exchange rate. Exchange rate fluctuation makes investment decisions and international trade more difficult due to the uncertainty and risk associated with increases in exchange rate volatility. The amount of international publications by Nigerian researchers and academics has drastically reduced in recent times due to increasing exchange rate volatility as more Nigerian currencies are needed in exchange of a small amount of US Dollars and other foreign currencies. This has hindered scientific communication, findings and exchange of ideas across the globe [1].

As argued by Hericourt and Poncet [1], exchange rate fluctuations impact negatively on firms exporting behaviour and rendered them financially vulnerable thereby affecting the financial development of such firms. Aghion et al. [2] provided supportive evidence on the negative impact of exchange rate volatility on economic growth in developing nations and emphasized how financial development reduces the negative effects of exchange rate volatility on economic development. Taiwo and Adesola [3] emphasized the need for a stable exchange rate in improving the lending ability of commercial banks to channel credit facilities to the economy for effective development. Given the high level of risk and uncertainty associated with exchange rate volatility and the frequent changes in exchange rate and policy of many developing countries including Nigeria, there is every need to obtain estimates of exchange rate volatility across time using accurate volatility measuring tools.

The Autoregressive Conditional Heteroskedasticity (ARCH) and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models developed by Engle [4] and Bollerslev [5] respectively have been found useful in modeling and forecasting volatility of financial time series data. While the symmetric ARCH and GARCH model are good at capturing volatility clustering, shocks persistence and other statistical regularities, they do not capture asymmetry and leverage effect in financial returns. The extensions of GARCH models called the asymmetric models such as the exponential GARCH (EGARCH) [6], threshold GARCH (TGARCH) [7], power GARCH (PGARCH) [8] and Glosten, Jaganathan and Runkle GARCH (GJR-GARCH) [9] among others are very useful in capturing asymmetry and leverage effect in financial return series.

Many scholars have applied both symmetric and asymmetric GARCH family models in modeling and forecasting exchange rate volatility across the globe both in developed and emerging economies and the empirical evidence are well documented in the literature. For example, [10] examined the performance of GARCH family models in forecasting the volatility behaviour of Pakistani FOREX market using daily FOREX rates data ranging from January 2001 to December 2009. The symmetric GARCH-M model found evidence of volatility clustering and persistence of shocks in the FOREX market. The result of the EGARCH showed evidence of asymmetry in volatility. Thorlie et al. [11] examined the accuracy and forecast performance ability of exchange rate return volatility for the Leones/USA dollars in ARMA-GARCH framework with normal and non-normal distributions using monthly exchange rate returns data from January 2004 to December 2013. They found that symmetric and asymmetric GARCH models performed better under non-normal distribution than the normal distribution and improved the overall estimation process for measuring conditional volatility in exchange rate returns. The GJR-GARCH model with skewed Student-t innovation was most successful and better in forecasting the Sierra Leone exchange rate volatility. The asymmetric GARCH models found evidence of asymmetry in exchange rate returns indicating presence of leverage effect. Omari et al. [12] employed GARCH-types models in modeling exchange rate volatility of the USD/KES exchange rate in Kenya using daily closing observations for the period January 3, 2003 to December 31, 2015. The study applied symmetric GARCH models such as GARCH (1, 1) and GARCH-M in capturing most of the stylized facts about exchange rate returns like volatility clustering and persistence as well as asymmetric models such as EGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1, 1) models that capture asymmetry and leverage effect. Results showed that asymmetric APARCH, GJR-GARCH and EGARCH models with Student-t innovation density were most adequate for estimating the volatility of the exchange rates in Kenya. See also several authors [13] and [14] for similar contributions.

In Nigeria, several documented evidence on exchange rate volatility modeling is also found in the literature. For example, [15] applied ARCH and GARCH models to examine the degree of volatility of USD/Naira exchange rate using monthly exchange rate data from 1986 to 2008. The result showed the presence of volatility clustering and over persistence of volatility shocks. Onakoya [16] employed asymmetric EGARCH model to investigate the relative contributions of stock market volatility on real output in Nigeria over the period from 1980 to 2010. The findings of the study revealed that volatility shock was quite persistent in Nigeria and this might distort economic growth of the country. Musa and Abubakar [17] in their study employed GARCH (1,1), GJR-GARCH (1,1), TGARCH (1,1) and TS-GARCH (1,1) models to investigate the volatility of daily closing US Dollar/Naira exchange rate data for the period 01/06/2000 to 26/07/2011 consisting of 4083 observations. Results revealed that apart from TGARCH (1,1) model which showed mean-reverting behaviour in the conditional variance, all the other GARCH models showed over persistence of volatility shocks indicating non-stationarity of the conditional variance processes. The GJR-GARCH (1,1) and TGARCH (1,1) models showed evidence for the existence of statistically significant asymmetric effect without leverage effect. TGARCH (1,1) and TS-GARCH (1,1) models were found superior over other GARCH models in explaining USD/Naira exchange rate volatility in Nigeria.

David et al. [18] examined the naira exchange rate against four foreign currencies: US Dollar, Euro, British Pound and Japanese Yen. The weekly data on these exchange rates spanned from January 2002 to May 2015. They employed lower symmetric and asymmetric GARCH specifications. Results of the symmetric models showed volatility persistence in all the foreign exchange rate returns. Results of the asymmetric model showed superior forecasting performance over symmetric GARCH with different impacts for both negative and positive volatility shocks. Emenike [19] employed symmetric GARCH (1,1) and asymmetric GJR-GARCH (1,1) models to estimate and compare the volatilities of official, interbank and bureaux de change markets Naira/US dollar exchange rates from January 1995 to December 2014. The results of the study showed evidence of volatility clustering in the interbank market and bureaux de change Naira/US dollar exchange rates. Volatility clustering and persistence was found to be higher in bureaux de change than in other exchange rates in Nigeria. more surveys are reported in the following papers [20,21,22,23,24,25].

From the foregoing, it is glaring to know that GARCH family models have been found useful by researchers around the world, including Nigeria in measuring exchange rate volatility. This study therefore, extends the existing literature by combining symmetric ARCH and GARCH models as well as asymmetric EGARCH and TGARCH models with heavy-tailed distributions in measuring Naira/US Dollar exchange rate volatility in Nigeria using more recent data.

2 Materials and Methods

2.1 Source of data and data integration

The data used in this research work are the daily closing Naira/US Dollar exchange rates for the period 12th November, 2001 to 12th January, 2017 making a total of 3665 observations. The data is obtained from Central Bank of Nigeria's website. The daily returns r_t were calculated as the continuously compounded returns corresponding to the first differences in logarithms of closing prices of successive days.

$$r_t = \log\left(\frac{S_t}{S_{t-1}}\right) \times 100 = [\log(S_t) - \log(S_{t-1})] \times 100 \tag{2.1}$$

where S_t denotes the closing market index at the current day (t) and S_{t-1} denotes the closing market index at the previous day $(t-1)$.

2.2 Jarque-Bera test of normality

Jarque and Bera [26,27] proposed a normality test which is goodness-of fit test of whether sample data have the skewness and kurtosis matching a normal distribution. The test is usually used to test the null hypothesis that the series is normally distributed. Given a return series $\{r_t\}$ the test statistic JB is defined as:

$$JB = \frac{T}{6}\left(S_k^2 + \frac{1}{4}(K_u - 3)^2\right) \tag{2.2}$$

where S_k is the sample skewness which is estimated by:

$$S_k = \frac{\mu_3}{\mu_2^{3/2}} = T^{1/2}\sum_{t=1}^{T}(r_t - \bar{r})^3 \Bigg/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{3/2} \tag{2.3}$$

and K_u is the sample kurtosis which is estimated by:

$$K_u = \frac{\mu_4}{\mu_2^2} = T\sum_{t=1}^{T}(r_t - \bar{r})^4 \Bigg/ \left(\sum_{t=1}^{T}(r_t - \bar{r})^2\right)^{2} \tag{2.4}$$

where T is the number of observations and $\bar{r}$ is the sample mean. The normal distribution has a skewness equal to 0 with a kurtosis of 3.

2.3 The Phillips-Perron (PP) unit root test

Phillips and Perron [28] propose an alternative non-parametric method of controlling for serial correlation when testing for a unit root. The PP method estimates the non-augmented Dickey-Fuller test equation

$$\Delta Y_t = \alpha Y_{t-1} + X_t^{'}\delta + \varepsilon_t \tag{2.5}$$

and modifies the $t-$ratio of the α coefficient so that serial correlation does not affect the asymptotic distribution of the test statistic. The PP test is based on the statistic:

$$\tilde{t}_\alpha = t_\alpha \left(\frac{\varphi_0}{\phi_0}\right)^{1/2} - \frac{T(\phi_0 - \varphi_0)(se(\hat{\alpha}))}{2\phi_0^{1/2} s} \quad (2.6)$$

where $\hat{\alpha}$ is the estimate of α, and t_α is the t −ratio for $\alpha, se(\hat{\alpha})$ is the standard error of $\hat{\alpha}$, and s is the standard error of the test regression, φ_0 is a consistent estimate of the error variance in (2.6) which is calculated as $(T-k)s^2/T$, where k is the number of regressors and ϕ_0 is an estimator of the residual spectrum at frequency zero.

2.4 Model specification

2.4.1 Autoregressive Conditional Heteroskedasticity (ARCH) model

The Autoregressive Conditional Heteroskedasticity (ARCH) model was first introduced by [4] to predict the conditional variance of return series. An ARCH (q) model is specified as:

$$r_t = \mu + \varepsilon_t; \;\; \varepsilon_t = e_t \sigma_t^2 \quad (2.7)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad (2.8)$$

ARCH (3) model is specified as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-3}^2 \quad (2.9)$$

where r_t is a return series, $\{e_t\}$ are i.i.d. random variables and σ_t^2 is a conditional variance of returns at time t which must be non-negative. The stationarity condition of ARCH (q) model is that $\sum \alpha_i < 1$.

2.4.2 Generalized ARCH (GARCH) model

The GARCH model was developed by Bollerslev [5], a former Ph.D student of Engle. Assuming a log return series $r_t = \mu + \varepsilon_t$ where ε_t is the error term at time t. The ε_t follows a GARCH (p, q) model if:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (2.10)$$

with constraints $\omega > 0, \alpha_i \geq 0, i = 1, 2, \ldots, q$ and $\beta_j \geq 0, j = 1,2, \ldots, p$; $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ to ensure conditional variance to be positive as well as stationary. The basic GARCH (1,1) model is given by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2.11)$$

The stationarity condition of a basic GARCH (1,1) is that the sum of ARCH and GARCH terms is strictly less than one (i.e., $\alpha_1 + \beta_1 < 1$).

2.4.3 The exponential GARCH (EGARCH) model

The EGARCH model was proposed by [6] to allow for asymmetric effects of positive and negative asset returns. It can be expressed as:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left\{\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right\} + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) + \sum_{k=1}^{r} \gamma_k \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] \quad (2.12)$$

Here γ represents the asymmetric coefficient in the model, β coefficient represents the measure of persistence. The conditional variance equation for EGARCH (1,1) model specification is given as:

$$\ln(\sigma_t^2) = \omega + \alpha_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \beta_1 \ln(\sigma_{t-1}^2) + \gamma \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] \quad (2.13)$$

2.4.4 Threshold GARCH (TGARCH) model

Threshold GARCH (TGARCH) was introduced independently by [7] and [9]. The generalized specification of TGARCH for the conditional variance is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{k=1}^{v} \gamma_k \varepsilon_{t-k}^2 I_{t-k}^- \quad (2.14)$$

where $I_t^- = 1$ if $\varepsilon_t < 0$ and 0 otherwise.

In this model, good news, $\varepsilon_{t-i} > 0$, and bad news, $\varepsilon_{t-i} < 0$, have differential effects on the conditional variance; good news has impact on α_i, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and we say that there is a leverage effect for the $i - th$ order. If $\gamma \neq 0$, the news impact is asymmetric.

The conditional variance equation for the TGARCH (1,1) model specification is given by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1}^- \quad (2.15)$$

2.5 Estimation of ARCH/GARCH models and innovation densities

In modelling the returns series for high-frequency financial time series, we obtain the estimates of ARCH/GARCH process by maximizing the likelihood function:

$$L\theta_t = -\,{}^{1}\!/_{2} \sum_{t-1}^{T} \left(\ln 2\pi + ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2} \right) \quad (2.16)$$

The Normal (Gaussian) Distribution is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty \quad (2.17)$$

The Student-t distribution is defined as:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \quad (2.18)$$

where v denotes the number of degrees of freedom and Γ denotes the Gamma function. The degree of freedom $v > 2$ controls the tail behaviour. The $t-$distribution approaches the normal distribution as $v \to \infty$.

The Generalized Error Distribution (GED) is given as:

$$f(z,\mu,\sigma,v)=\frac{\sigma^{-1}ve^{\left(-\frac{1}{2}\left|\frac{\left(\frac{z-\mu}{\sigma}\right)}{\lambda}\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma\left(\frac{1}{v}\right)},1<z<\infty \tag{2.19}$$

$v>0$ is the degrees of freedom or tail -thickness parameter and $\lambda=\sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right)/\Gamma\left(\frac{3}{v}\right)}$. If $v=2$, the GED yields the normal distribution. If $v<1$, the density function has thicker tails than the normal density function, whereas for $v>2$ it has thinner tails.

2.6 Model selection criteria

To select the best fitting ARCH/GARCH model, Akaike Information Criterion (AIC) [29], Schwarz Information Criterion (SIC) [30] and Hannan-Quinn Information Criterion (HQC) [31] and Log likelihood are the most commonly used model selection criteria. These criteria were used in this study and are computed as follows:

$$AIC(K)=-2\ln(L)+2K \tag{2.20}$$

$$SIC(K)=-2\ln(L)+Kln(T) \tag{2.21}$$

$$HQC(K)=2\ln[\ln(T)]\,K-2\ln\,(L) \tag{2.22}$$

where K is the number of independently estimated parameters in the model, T is the number of observations; L is the maximized value of the Log- Likelihood for the estimated model and is defined by:

$$\left.\begin{aligned} L&=\prod_{i=0}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}exp\left[-\sum_{i=1}^{n}\frac{\left(y_i-f(x)\right)^2}{2\sigma_i^2}\right]\\ \text{Log}\,L&=In\left[\prod_{i=1}^{n}\left(\frac{1}{2\pi\sigma_i^2}\right)^{1/2}\right]-\frac{1}{2}\sum_{i=1}^{n}\frac{\left(y_i-f(x)\right)^2}{\sigma_i^2}\end{aligned}\right\} \tag{2.23}$$

Thus given a set of estimated ARCH/GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and larger log likelihood value.

3 Results and Discussion of Empirical Findings

The following subsections present the results and discussion of data analysis in this study.

3.1 Descriptive statistics of exchange rate returns

To better understand the distributional characteristics of the exchange rate log returns, we compute the descriptive statistics and the result is reported in Table 1.

Table 1. Summary statistics of Naira/US Dollar exchange rate Log returns

N	Mean	Max.	Min.	SD	Skew.	Kurt.	JB	P-value
3665	0.0272	230.90	-231.02	5.7282	0.0275	1452.45	3.21E+08	0.0000

The summary statistics reported in Table 1 reveals that the average daily Naira/US Dollar exchange rate log return is 0.0272% with a daily standard deviation of 5.728%. This reflects a high level of dispersion from the average log returns in the economy over the period under review. The wide gap between the maximum returns of 230.90 and minimum returns of -231.02 give supportive evidence to the high level of variability of Naira/US Dollar exchange rate changes in Nigeria over the study period. The high kurtosis value of 1452.452 suggests that big shocks of either sign are more likely to be present in the Naira/US Dollar exchange rate series and that the returns series are clearly leptokurtic. Similarly, the skewness coefficient of 0.0275 indicates evidence of asymmetry. The null hypothesis of zero skewness and kurtosis coefficient of 3 are rejected at 1% significance levels suggesting that the daily Naira/US Dollar exchange rate log returns series do not follow a normal distribution. This rejection of normality in the returns series is confirmed by Jarque-Bera test as its associated p-value is far below 1% marginal significance level.

3.2 Phillips-Perron unit root test result

To examine the unit root and stationarity characteristics of the daily Naira/US Dollar exchange rate series and its log return, we employ Phillips and Perron unit root test. The test is conducted for both constant only and for constant and linear trend. The result of the PP test is reported in Table 2.

Table 2. Phillips and Perron unit root test result

Variable	Option	PP test statistic	P-value	Critical value		
				1%	5%	10%
Y_t	Constant only	-0.7669	0.8276	-3.4320	-2.8621	-2.5671
	Constant and trend	-2.2687	0.4505	-3.9606	-3.4110	-3.1273
r_t	Constant only	-29.0168	0.0000**	-3.4320	-2.8621	-2.5671
	Constant and trend	-29.0580	0.0000**	-3.9606	-3.4110	-3.1273

Note: *** denotes the significant of PP test statistic at 1% marginal significance level.*

From the PP unit root test result presented in Table 2, we fail to reject the null hypothesis of unit root in the daily Naira/US Dollar exchange rate series at all conventional test sizes but the null hypothesis of PP unit root is rejected in the log returns at 1%, 5% and 10% significance levels both for constant only and for constant and linear trend. This means that the daily Naira/US Dollar exchange rate series are non-stationary (contains unit root) while their log returns are stationary.

We also test for the ARCH effects in the residuals of the returns using Engle's Lagrange Multiplier ARCH test and the result is presented in Table 3.

Table 3. Heteroskedasticity test for presence of ARCH effect

Lag	F-statistic	P-value	nR^2	P-value
1	2.5645	0.0051	2.5629	0.0059
30	0.2192	0.0006	0.2195	0.0005

The Engle's Lagrange Multiplier ARCH test at 5 percent significance level with up to 30 lags corresponding to one month trading period reported in Table 3 rejects the null hypothesis of no ARCH effects in the log returns. This means that the variances of log returns are heteroskedastic and suggests the use of ARCH/GARCH model for capturing the time varying volatility in the log returns.

3.3 Searching for optimal symmetric and asymmetric GARCH-type models

In order to select optimal symmetric and asymmetric ARCH/GARCH models that best fit the Naira/US Dollar exchange rate log returns series, we use the log-likelihoods in conjunction with some selected information criteria. The best fitting model is one with highest log-likelihood and lowest information criteria. The results are reported in Table 4.

Table 4. ARCH/GARCH model order selection using Log likelihood and information criteria

Model	Distribution	LogL	AIC	SC	HQC
ARCH (1)	ND	-10391	5.6736	5.6787	5.6754
ARCH (2)	ND	-10455	5.7089	5.7157	5.7113
ARCH (3)	ND	-10482	5.7244	5.7329	5.7274
ARCH (1)	STD	-7308	3.9925	4.0027	3.9962
ARCH (2)	STD	3743	-2.0403	-2.0319	-2.0373
ARCH (3)	STD*	11787	-6.4317	-6.4248	-6.4292
ARCH (1)	GED	-7420	4.0526	4.0593	4.0550
ARCH (2)	GED	-10696	5.8411	5.8496	5.8442
ARCH (3)	GED	-10705	5.8466	5.8568	5.8502
GARCH (1,1)	ND	-10993	6.0027	6.0095	6.0051
	STD*	-6753	3.6888	3.6973	3.0918
	GED	-7977	4.3568	4.3653	4.3598
EGARCH (1,1)	ND	-9539	5.2098	5.2183	5.2128
	STD	-8624	5.1626	5.1524	5.1589
	GED	-6391	3.4917	3.5018	3.4953
TGARCH (1,1)	ND	-10797	5.8965	5.9049	5.8995
	STD*	-5865	3.2049	3.2150	3.2085
	GED	-7379	4.0314	4.0415	4.0350

***Note**: *denotes model selected by information criteria*

From the results of Table 4 we observe that out of the 18 contesting GARCH-type models estimated for the exchange rate log returns using different innovations, symmetric ARCH (3), GARCH (1,1) and asymmetric TGARCH (1,1) all with student-t innovations and asymmetric EGARCH (1,1) model with GED distribution have the highest log-likelihoods and lowest information criteria and are therefore selected as the best fitting models to describe the volatility of Naira/USD exchange rate log returns in Nigeria under the review period.

3.4 Parameter estimates of symmetric ARCH and GARCH models

The parameter estimates of symmetric ARCH (3) and basic GARCH (1,1) models with student-t distribution are presented in Table 5.

Table 5. Parameter estimates of symmetric GARCH models with Student- t innovations

Variable	Coefficient	Std. error	z-Statistic	P-value
ARCH (3) model				
ω	8.438141	0.264812	31.86466	0.0000
α_1	1.440828	0.776574	1.855365	0.0035
α_2	-0.009078	0.001581	-5.741353	0.0000
α_3	0.008932	0.001550	5.762329	0.0000
v	16.78674	0.371049	45.24130	0.0000
ϕ	1.440682	ARCH LM Test	0.000203	0.9886
GARCH (1,1) model				
ω	6.644938	0.212313	31.29790	0.0000
α_1	0.312924	0.025389	4.447809	0.0000
β_1	0.801102	0.000253	-4.348945	0.0000
v	11.79244	0.300429	39.25205	0.0000
λ	1.114026	ARCH LM Test	0.002694	0.9586

Note: $\phi = \alpha_1 + \alpha_2 + \alpha_3$ and $\lambda = \alpha_1 + \beta_1$ measures the volatility shock persistence

From the result of the upper panel of Table 5, the ARCH (3) model estimates are presented in equation (3.1) below:

$$\sigma_t^2 = 8.438141 + 1.440828\varepsilon_{t-1}^2 - 0.009078\varepsilon_{t-2}^2 + 0.008932\varepsilon_{t-3}^2 \quad (3.1)$$

The parameters of the model are all statistically significant and the ARCH (3) model have captured all the ARCH effects as the p-value of the ARCH LM test is highly statistically insignificant. However, the ARCH (3) model as presented by equation (3.1) is unstable as the sum of ARCH terms is greater than unity (i.e., $\alpha_1 + \alpha_2 + \alpha_3 = 1.440682 > 1$). This means that the conditional variance is unstable and the process is non-stationary which can eventually explodes to infinity.

From the result of symmetric GARCH (1,1) model presented in the lower panel of Table 5, the GARCH (1,1) model estimates are presented in equation (3.2) below:

$$\sigma_t^2 = 6.644938 + 0.312924\varepsilon_{t-1}^2 + 0.801102\sigma_{t-1}^2 \quad (3.2)$$

The parameters of the basic GARCH (1,1) model in the variance equation are all significant at 1% marginal significance levels. The significant of α_1 (ARCH (1) term) shows that the volatility of risk is affected by past square residual terms. The significant of β_1 (GARCH (1) term) shows that past volatility of exchange rate affects current volatility. The stationarity condition of the model is not satisfied since the sum of ARCH and GARCH terms is greater than 1(i.e., $\alpha_1 + \beta_1 = 1.114026 > 1$). This shows that the conditional variance of the exchange rate log returns is unstable and the entire process is non-stationary which may eventually explodes to infinity. The GARCH (1,1) model captured all the ARCH effects in the log return series as shown by the non-significant p-value of the ARCH LM test reported in Table 5.

3.5 Result of asymmetric EGARCH (1,1) and TGARCH (1,1) models

The parameter estimates of asymmetric EGARCH (1,1) and TGARCH (1,1) models are presented in Table 6.

Table 6. Parameter estimates of asymmetric EGARCH (1,1) and TGARCH (1,1) models

Parameter	Coefficient	Std. error	z-Statistic	P-value
EGARCH (1,1) Model with GED innovation				
ω	2.210249	0.255024	8.666819	0.0000
α_1	0.256155	0.000553	45.48765	0.0000
β_1	0.851576	0.173506	-2.948456	0.0032
γ	-0.1017162	0.000557	-30.81541	0.0000
v	1.368214	0.003853	355.1076	0.0000
λ	1.107731	ARCH LM Test	0.000776	0.9778
TGARCH (1,1) Model with student-t innovation				
ω	4.312117	0.172987	24.92744	0.0000
α_1	0.477004	0.026952	2.857058	0.0043
β_1	0.901948	0.000663	-2.939332	0.0033
γ	0.035521	0.028634	-1.240485	0.0048
v	8.371334	0.244898	34.18288	0.0000
$\alpha_1 + \beta_1 + \gamma/2$	1.3611915	ARCH LM Test	0.002909	0.9570

Note: $\lambda = \alpha_1 + \beta_1$ measures the volatility shock persistence

The result at the upper panel of Table 6 shows the estimates of asymmetric EGARCH (1,1) model parameters with GED distribution on the daily Naira/US dollar exchange rate log returns as presented by equation (3.3) while the result in the lower panel of Table 6 depicts the estimates of asymmetric TGARCH (1,1) model parameters on the daily Naira/US dollar exchange rate log returns as represented by equation (3.4)

$$\ln(\sigma_t^2) = 2.210249 + 0.256155\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + 0.851576\ln(\sigma_{t-1}^2) - 0.1017162\left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] \quad (3.3)$$

$$\sigma_t^2 = 4.312117 + 0.477004\varepsilon_{t-1}^2 + 0.901948\sigma_{t-1}^2 + 0.035521\varepsilon_{t-k}^2 \quad (3.4)$$

All parameters of the estimated models are significant. The significance of the ARCH (1) and GARCH (1) terms indicates that previous square error terms significantly affects volatility and that past volatility of exchange rate also influences current volatility. The asymmetric and leverage effect parameters for the EGARCH (1,1) and TGARCH (1,1) are ($\gamma = -0.1017162$) and ($\gamma = 0.035521$) respectively. The negative and significant value of γ for EGARCH and the positive and significant value of γ for TGARCH suggest evidence for the existence of asymmetry and leverage effects. These results indicate that bad news (negative shocks) increases volatility more than good news (positive shocks) of the same magnitude. However, EGARCH (1,1) and TGARCH (1,1) models are non-stationary as the sum of ARCH and GARCH terms are greater than unity in both models. This shows over persistence of volatility shocks and that the conditional variances are unstable and the Naira/US Dollar exchange rate log return is unpredictable in Nigerian foreign exchange market. The EGARCH (1,1) and TGARCH (1,1) models have sufficiently captured all the ARCH effects in the Naira/US Dollar exchange rate log return series as shown by the non-significant p-values of the ARCH LM test reported in Table 6.

3.6 Models implication for investors and policy makers

From the results of symmetric ARCH (3), basic GARCH (1,1) asymmetric EGARCH (1,1) and TGARCH (1.1) models presented in this work, it is clearly seen that all the estimated volatility models are associated with over persistence of shocks which means that the conditional variance of the Naira/US Dollar exchange rate log returns is unstable and can eventually explode to infinity. High volatility shocks persistence is associated with high level of market risk. The implication is that investors can only buy and sell stocks as soon as they are purchased while keeping stocks for a long period of time involves high level of market risk. This is so because in a conditional variance process with high persistence of shocks, investors can gain or loss indefinitely. Also, stocks with high shock persistence are associated with long memory and possibly contaminated with sudden shifts in variance which is bounds to affect investors' decisions negatively.

4 Conclusion

In this paper, attempt has been made to identify and model the daily exchange rate log returns of Nigerian Naira against United States Dollar. The data comprised daily exchange rates of Naira/US Dollar for the period 12th November, 2001 to 12th January, 2017. Jarque-Bera test of normality was used to examine the normality characteristics of returns while the stationarity properties of the data were investigated using Phillips-Perron unit root test. Symmetric ARCH and basic GARCH models, as well as asymmetric EGARCH and TGARCH models with non-Gaussian errors were employed to analyze the daily exchange rate log returns. Log likelihood and information criteria such as Akaike information criterion (AIC), Schwartz information criterion (SIC) and Hannan Quinn criterion (HQC) were used to select the optimal and best fitting model among the competing GARCH models.

Results showed that symmetric ARCH (3) and basic GARCH (1,1) with student-t innovation as well as asymmetric EGARCH (1,1) model with GED distribution and TGARCH (1,1) model with student-t distribution were the best fitting models for the exchange rate log return series. The conditional variances of all the estimated GARCH models were found to be unstable with higher volatility shock persistence. The asymmetric EGARCH (1,1) and TGARCH (1,1) models showed supportive evidence for the existence of asymmetry with leverage effects indicating that negative shocks increase volatility more than positive shocks of the same sign. This results signaled potential loss or gain with higher level of risk and uncertainty in Nigerian foreign exchange market. Traders and investors in Nigerian foreign exchange market are therefore advised to buy or sell stocks as soon as they are purchased bearing in mind the level of risk associated with keeping stocks for a long period of time. As a policy implication, this study recommends that the Nigerian foreign exchange market be made less volatile by allowing aggressive and excessive trading which increases market depth to persist.

Competing Interests

Authors have declared that no competing interests exist.

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