Asian Journal of Research and Reviews in Physics

5(1): 1-11, 2021; Article no.AJR2P.73259
ISSN: 2582-5992

An Analysis for Conservative and Non-conservative *f(R, T)* Gravity Models

Diyadin Can[1] and Ertan Güdekli[1*]

[1]Department of Physics, Istanbul University, Turkey.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJR2P/2021/v5i130152
Editor(s):
(1) Dr. Khalil Kassmi, Mohamed Premier University, Morocco.
(2) Prof. Shi-Hai Dong, National Polytechnic Institute, Mexico.
(3) Dr. Sebahattin Tüzemen, Atatürk University, Turkey.
Reviewers:
(1) Sukadev Sahoo, India.
(2) HOVA Hoavo, University of Kara, Togo.
(3) Mustafa SALTI, Mersin University, Turkey.
(4) Maryam Roushan, University of Mazandaran, Iran.
(5) Javier Joglar Alcubilla, Spain.
Complete Peer review History: https://www.sdiarticle4.com/review-history/73259

Original Research Article

Received 29 June 2021
Accepted 09 September 2021
Published 11 September 2021

ABSTRACT

As it is known that General Theory of Relativity does not explain the current acceleration of the universe, so there are many attempts to generalize this theory in order to explain the cosmic acceleration without introducing some dark components such as the Dark Energy. Because of the crowd of models in literature, a need to check the models according to some criteria arises. In this study, we analyze two classes of models by means of energy condition restrictions and illustrate the analysis of those classes by graphical simulations. We consider the conservative and non-conservative cases of two classes of $f(R,T)$ models to perform the analysis. The results of the viability of the classes are discussed and it is found that the value of the Hubble constant has no effect on the viability of the models. Focusing on some general classes for the models, we restrict them by means of the so-called energy conditions the energy-momentum tensor on physical grounds. Besides, we find numerical values for coefficients of those classes of models.

Keywords: General theory of relativity; modified gravity models; cosmological models; f(R,T) gravity.

**Corresponding author: E-mail: gudekli@istanbul.edu.tr;*

1. INTRODUCTION

Among the most interesting pursuits of cosmology today are the dark components, which are dark energy (DE) and dark matter (DM) [1-6]. DE is a mechanism thought to exist in addition to the standard matter energy content to explain the accelerated expansion process in the late time era of the universe, that is, since the last four billion years. It is thought that the current ingredients of the universe in terms of mass energy content, quantitatively, should be 5% baryonic matter, 27% dark matter and 68% dark energy [7]. Observations such as Type-Ia supernova observations, cosmic background microwave anomalies, large-scale structure of the universe, baryon acoustic oscillations, weak lensing indicate cosmic acceleration [8-11]. The observational discovery of the advanced acceleration of the universe has led to interesting explanations that continue to this day.

Instead of making various assumptions about the matter energy content on the right side of Einstein's Field Equations (EFE), $G_{\mu\nu} = \kappa T_{\mu\nu}$, to explain the DE and DM issues, as an alternative, it was considered to modify Einstein's gravitation theory by changing the geometric part on the left side of the equation. These changes are generally referred to as corrections to the Theory of General Relativity (GR). They are considered in two classes as early epoch (ultraviolet corrections inspired from the electromagnetic spectrum) and late epoch corrections (infrared corrections) in terms of phases. As a result of replacing the Ricci scalar R with an arbitrary $f(R)$ function in the Einstein-Hilbert action, $S_{EH} = \frac{1}{2\kappa}\int R\sqrt{-g}d^4x$, it is possible to arrive at a new theory of gravity, which can be an alternative to the GR. It is also possible to obtain new gravitational theories by using the curvature invariants, which are the combinations of Ricci curvature tensor R_{ab}, Riemann curvature tensor R_{abcd}, and Weyl tensor C_{abcd}, as arguments of the function f. The theories $f(G)$, $f(R,G)$, $f(R,T)$, and $f(G,T)$, where the Gauss-Bonnet invariant that is defined as $G = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$, and the trace of energy-momentum tensor $T = g^{ab}T_{ab}$ are used, are some of the examples. These gravitational theories which are established as alternatives to GR are called modified gravitational theories [12-18]. In this paper, we study two types of $f(R,T)$ gravity models and will study some other f theories in future papers.

While EFE are quadratic, modified gravity theories generally lead to fourth-order differential equations. Because of their higher degrees of freedom, such theories are mathematically more complicated than GR, but they offer much richer possibilities when dealing with DE, DM and other cosmological issues. Modification of the GR to four or more dimensions by using such curvature invariants was attempted long before the DE issue. The inadequacies of classical GR when it comes to strong gravitational fields, on the one hand, and the efforts to quantify gravity on the other hand, necessitated importing such invariants.

In this work, we first obtain the field equations for $f(R,T)$ models (In fact we reduce the more general equations of $f(R,G,T)$ gravity models which we have already obtained [19] and will be discussed in upcoming papers). Those field equations are introduced for the both conservation and non-conservation cases. In each case the Lagragian in two choices as $L_m = p^m$ and $L_m = -\mu^m$ generates a specific set of field equations. And, within each choice we represent the equations as two different interpretations which are called 1st and 2nd INTERPRETATIONs. After these general discussions we analyze the viability of two sets of $f(R,T)$ gravity models in the point of view of cosmological constraints. We take into account two models linear in R and vary by different ways with respect to T such as $f(R,T) = R + K_1 \cdot \ln(-T)$ and $f(R,T) = R + K_2 \cdot (-T)^\gamma$. The object of this work is analysing those models by considering the energy conditions and then discussing the models to find out the viable values for the coefficients, powers and constants.

2. GENERAL DISCUSSIONS

We split the Lagrangian into two interpretations as 1st INTERPRETATION and 2nd INTERPRETATION, and consider two situations of the standard matter as conservation and nonconservation cases. In the formulae given below w, w_{DE}, Ω_m, $\Omega_m^{t.eff}$, Ω_g, Ω_P, $\Omega_P^{t.eff}$, j, s, H, q represent state parameter of ordinary matter, equation of state of dark energy, ordinary matter density parameter, total effective matter density parameter, curvature parameter, pressure parameter, effective pressure density parameter, jerk parameter, snap parameter, Hubble parameter and deceleration parameter, respectively while '0' in the subscripts denote their current values. F, r, g, and t represent the dimensionless forms of the function f and the parameters R, G, and T. The dimensionless parameters $\Re, \Gamma$ and τ which are defined in the text are also used in the formulae. The subscripts denote the derivatives with respect to the dimensionless parameters, such as $\frac{\partial F}{\partial r} \equiv F_r$, $\frac{\partial^2 F}{\partial r^2} \equiv F_{rr}$, and for the current values of them, i.e., $\left(\frac{\partial F}{\partial r}\right)|_{t=0} \equiv F_{r,0}$ and so on. Furthermore we illustrate derivatives of R, G, T parameters with recpect to time by $\dot{R}, \dot{G}$ and $\dot{T}$, while dimensionless forms of them are obtained by $\Re = \left(\frac{c^2}{H^2}\right) R$, $\Re^* = \left(\frac{c^2}{H^3}\right) \dot{R}$, $\Re^{**} = \left(\frac{c^2}{H^4}\right) \ddot{R}$, $\Gamma = \left(\frac{c^4}{H^4}\right) G$, $\Gamma^* = \left(\frac{c^4}{H^5}\right) \dot{G}$, $\Gamma^{**} = \left(\frac{c^4}{H^6}\right) \ddot{G}$, $\tau = \left(\frac{\kappa^2 c^2}{3H^2}\right) T$, $\tau^* = \left(\frac{\kappa^2 c^2}{3H^3}\right) \dot{T}$, $\tau^{**} = \left(\frac{\kappa^2 c^2}{3H^4}\right) \ddot{T}$ where c is the speed of light, κ^2 is the Einstein coupling constant, R is the Ricci scalar curvature, G is the Gauss-Bonnet term, T is the trace of energy-momentum tensor. With the selection of the Lagrange matter density as $L_m = p^m$, let us collectively give the general formulas;

$$\tau_0 = (-1 + 3w)\Omega_{m,0} \tag{1}$$

$$\tau_0^* = \frac{-2(1+w)\left[3\left(3 + F_{t,0}\right) + F_{tr,0}\Re_0^* + F_{tg,0}\Gamma_0^*\right]}{6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0}\tau_0 \tag{2}$$

$$\begin{aligned}
\tau_0^{**} = {} & \frac{-2(1+w)}{\left[6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0\right]^2}\Big\{\big\{\big[-3(1+q_0)\left(3 + F_{t,0}\right) \\
& + 3\left(F_{tr,0}\Re_0^* + F_{tg,0}\Gamma_0^* + F_{tt,0}\tau_0^*\right) + \left(F_{trr,0}\Re_0^{*\,2} + F_{trg,0}\Re_0^*\Gamma_0^* + F_{trt,0}\Re_0^*\tau_0^*\right) \\
& + \left(F_{tgr,0}\Gamma_0^*\Re_0^* + F_{tgg,0}\Gamma_0^{*\,2} + F_{tgt,0}\Gamma_0^*\tau_0^* + F_{tr,0}\Re_0^{**} + F_{tg,0}\Gamma_0^{**}\right)\big]\tau_0 \\
& + \left[3\left(3 + F_{t,0}\right) + F_{tr,0}\Re_0^* + F_{tg,0}\Gamma_0^*\right]\tau_0^*\big\}\left[6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0\right] \\
& - \big[(3-w)\left(F_{tr,0}\Re_0^* + F_{tg,0}\Gamma_0^* + F_{tt,0}\tau_0^*\right) \\
& + 2(1+w)\left(F_{ttr,0}\Re_0^* + F_{ttg,0}\Gamma_0^* + F_{ttt,0}\tau_0^*\right)\tau_0 + 2(1+w)F_{tt,0}\tau_0^*\big]\big[3\left(3 + F_{t,0}\right) \\
& + F_{tr,0}\Re_0^* + F_{tg,0}\Gamma_0^*\big]\tau_0\Big\}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\Omega_{m,0}^{t.eff} = {} & \frac{1}{F_{r,0}}\Big[\Omega_{m,0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} - \frac{1}{6}\left(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0\right) - \left(F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*\right) \\
& - 4\left(1 - \Omega_{k,0}\right)\left(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*\right)\Big]
\end{aligned} \tag{4}$$

$$\begin{aligned}
\Omega_{P,0}^{t.eff} = {} & \frac{1}{F_{r,0}}\Big\{w\Omega_{m,0} + \frac{1}{6}\left(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0\right) + \frac{2}{3}\left(F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*\right) \\
& + \frac{1}{3}\big[F_{rrr,0}\Re_0^{*\,2} + F_{rgg,0}\Gamma_0^{*2} + F_{rtt,0}\tau_0^{*\,2} + 2\left(F_{rrg,0}\Re_0^*\Gamma_0^* + F_{rrt,0}\Re_0^*\tau_0^* + F_{rgt,0}\Gamma_0^*\tau_0^*\right) \\
& + F_{rr,0}\Re_0^{**} + F_{rg,0}\Gamma_0^{**} + F_{rt,0}\tau_0^{**}\big] - \frac{8}{3}q_0\left(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*\right) \\
& + \frac{4}{3}\left(1 - \Omega_{k,0}\right)\big[F_{grr,0}\Re_0^{*\,2} + F_{ggg,0}\Gamma_0^{*\,2} + F_{gtt,0}\tau_0^{*\,2} + 2F_{grg,0}\Re_0^*\Gamma_0^* + 2F_{grt,0}\Re_0^*\tau_0^* \\
& + 2F_{ggt,0}\Gamma_0^*\tau_0^* + F_{gr,0}\Re_0^{**} + F_{gg,0}\Gamma_0^{**} + F_{gt,0}\tau_0^{**}\big]\Big\}
\end{aligned} \tag{5}$$

where:

$$\begin{aligned}
\Re_0 &= 6\left(-q_0 + 1 - \Omega_{k,0}\right) \\
\Re_0^* &= 6\left(-q_0 + j_0 - 2 + 2\Omega_{k,0}\right) \\
\Re_0^{**} &= 6\left[q_0^2 + 8q_0 + 6 + s_0 - 2\Omega_{k,0}(q_0 + 3)\right] \\
\Gamma_0 &= -24q_0\left(1 - \Omega_{k,0}\right) \\
\Gamma_0^* &= 24\left[2q_0^2 + 3q_0 + j_0 - (j_0 + 3q_0)\Omega_{k,0}\right] \\
\Gamma_0^{**} &= 24\left[-2q_0^3 - 15q_0^2 - 12q_0 - 6q_0j_0 - 6j_0 + s_0 + (3q_0^2 + 12q_0 + 6j_0 - s_0)\Omega_{k,0}\right] \\
\tau_0 &= (-1 + 3w)\Omega_{m,0}
\end{aligned} \tag{6}$$

On the other hand, for the sake of simplicity, we take the dimensionless variables $r = \Re_0, g = \Gamma_0, t = \tau_0$ to analyze the models, in the softwares.

$$F_0 \equiv F|_{t=t_0} \equiv F|_{r=\Re_0, g=\Gamma_0, t=\tau_0} \equiv F(\Re_0, \Gamma_0, \tau_0)$$
$$F_{r..,0} \equiv F_{r..}|_{t=t_0} \equiv F|_{r=\Re_0, g=\Gamma_0, t=\tau_0} \equiv F_{r..}(\Re_0, \Gamma_0, \tau_0) \tag{7}$$

For $L_m = p^m$, 2nd INTERPRETATION: the equations (2) and (3) takes the following forms while we choose $H = H_0$;

$$\Omega_{m,0}^{t.eff} = \Omega_{m,0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} + (1-F_r)(1-\Omega_{k,0}) - \frac{1}{6}(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0) - (F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) - 4(1-\Omega_{k,0})(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*) \tag{8}$$

$$\begin{aligned}\Omega_{P,0}^{t.eff} = {} & w\Omega_{m,0} + \frac{1}{3}(1-F_{r,0})(2q_0 - 1 + \Omega_{k,0}) + \frac{1}{6}(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0) \\ & + \frac{2}{3}(F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) \\ & + \frac{1}{3}\big[(F_{rrr,0}\Re_0^{*2} + F_{rgg,0}\Gamma_0^{*2} + F_{rtt,0}\tau_0^{*2}) + 2(F_{rrg,0}\Re_0^*\Gamma_0^* + F_{rrt,0}\Re_0^*\tau_0^* + F_{rgt,0}\Gamma_0^*\tau_0^*) \\ & + F_{rr,0}\Re_0^{**} + F_{rg,0}\Gamma_0^{**} + F_{rt,0}\tau_0^{**}\big] - \frac{8}{3}q_0(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*) \\ & + \frac{4}{3}(1-\Omega_{k,0})\big[F_{grr,0}\Re_0^{*2} + F_{ggg,0}\Gamma_0^{*2} + F_{gtt,0}\tau_0^{*2} \\ & + 2(F_{grg,0}\Re_0^*\Gamma_0^* + F_{grt,0}\Re_0^*\tau_0^* + F_{ggt,0}\Gamma_0^*\tau_0^*) + F_{gr,0}\Re_0^{**} + F_{gg,0}\Gamma_0^{**} \\ & + F_{gt,0}\tau_0^{**}\big]\end{aligned} \tag{9}$$

as well as τ^* and τ^{**} does not change.

For $L_m = -\mu^m$, 1st and 2nd INTERPRETATIONs respectively,

$$\tau^* = \frac{-3(1+w)(3+F_t)}{6+(1-3w)F_t}\tau \tag{10}$$

$$\tau^{**} = \frac{-6(1+w)(3+F_t)}{[6+(1-3w)F_t]^2}\left\{-(1+q_0)[6+(1-3w)F_t] - 6(1+w)(3+F_t) + \frac{3(1+3w)(F_{tr}\Re^* + F_{tg}\Gamma^* + F_{tt}\tau^*)}{(3+F_t)}\right\}\tau \tag{11}$$

$$\Omega_{m,0}^{t.eff} = \frac{1}{F_{r,0}}\Big[\Omega_{m,0} - \frac{1}{6}(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0) - (F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) - 4(1-\Omega_{k,0})(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*)\Big] \tag{12}$$

$$\begin{aligned}\Omega_{P,0}^{t.eff} = \frac{1}{F_{r,0}}\Big\{ & w\Omega_{m,0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} + \frac{1}{6}(F_0 - F_{r,0}\Re_0 - F_{g,0}\Gamma_0) \\ & + \frac{2}{3}(F_{rr,0}\Re_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) \\ & + \frac{1}{3}\big[F_{rrr,0}\Re_0^{*2} + F_{rgg,0}\Gamma_0^{*2} + F_{rtt,0}\tau_0^{*2} + 2(F_{rrg,0}\Re_0^*\Gamma_0^* + F_{rrt,0}\Re_0^*\tau_0^* + F_{rgt,0}\Gamma_0^*\tau_0^*) \\ & + F_{rr,0}\Re_0^{**} + F_{rg,0}\Gamma_0^{**} + F_{rt,0}\tau_0^{**}\big] - \frac{8}{3}q_0(F_{gr,0}\Re_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*) \\ & + \frac{4}{3}(1-\Omega_{k,0})\big[F_{grr,0}\Re_0^{*2} + F_{ggg,0}\Gamma_0^{*2} + F_{gtt,0}\tau_0^{*2} \\ & + 2(F_{grg,0}\Re_0^*\Gamma_0^* + F_{grt,0}\Re_0^*\tau_0^* + F_{ggt,0}\Gamma_0^*\tau_0^*) + F_{gr,0}\Re_0^{**} + F_{gg,0}\Gamma_0^{**} \\ & + F_{gt,0}\tau_0^{**}\big]\Big\}\end{aligned} \tag{13}$$

$$\Omega_{m,0}^{t.eff} = \Omega_{m,0} + (1 - F_{r,0})(1 - \Omega_{k,0}) - \frac{1}{6}(F_0 - F_{r,0}\mathfrak{R}_0 - F_{g,0}\Gamma_0) - (F_{rr,0}\mathfrak{R}_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) - 4(1 - \Omega_{k,0})(F_{gr,0}\mathfrak{R}_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*) \tag{14}$$

$$\begin{aligned}\Omega_{P,0}^{t.eff} = {} & w\Omega_{m,0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} + \frac{1}{3}(1 - F_{r,0})(2q_0 - 1 + \Omega_{k,0}) + \frac{1}{6}(F_0 - F_{r,0}\mathfrak{R}_0 - F_{g,0}\Gamma_0) \\ & + \frac{2}{3}(F_{rr,0}\mathfrak{R}_0^* + F_{rg,0}\Gamma_0^* + F_{rt,0}\tau_0^*) \\ & + \frac{1}{3}\left[F_{rrr,0}\mathfrak{R}_0^{*2} + F_{rgg,0}\Gamma_0^{*2} + F_{rtt,0}\tau_0^{*2} + 2(F_{rrg,0}\mathfrak{R}_0^*\Gamma_0^* + F_{rrt,0}\mathfrak{R}_0^*\tau_0^* + F_{rgt,0}\Gamma_0^*\tau_0^*)\right. \\ & \left. + F_{rr,0}\mathfrak{R}_0^{**} + F_{rg,0}\Gamma_0^{**} + F_{rt,0}\tau_0^{**}\right] - \frac{8}{3}q_0(F_{gr,0}\mathfrak{R}_0^* + F_{gg,0}\Gamma_0^* + F_{gt,0}\tau_0^*) \\ & + \frac{4}{3}(1 - \Omega_{k,0})\left[F_{grr,0}\mathfrak{R}_0^{*2} + F_{ggg,0}\Gamma_0^{*2} + F_{gtt,0}\tau_0^{*2}\right. \\ & + 2(F_{grg,0}\mathfrak{R}_0^*\Gamma_0^* + F_{grt,0}\mathfrak{R}_0^*\tau_0^* + F_{ggt,}\Gamma_0^*\tau_0^*) + F_{gr,0}\mathfrak{R}_0^{**} + F_{gg,0}\Gamma_0^{**} \\ & \left. + F_{gt,0}\tau_0^{**}\right]\end{aligned} \tag{15}$$

$$w_{DE} = \frac{\Omega_{P,0}^{t.eff} - \Omega_{P,0}}{\Omega_{m,0}^{t.eff} - \Omega_{m,0}} \tag{16}$$

(1) - (15) written for $f(R,G,T) - gravity$ reduced to $f(R,T) - gravity$. If the terms Γ_0, Γ_0^*, Γ_0^{**} which are related to G, and the terms $F_{g,0}$, $F_{rg,0}$, ... which include derivative terms with respect to g are removed the reduced equations for $f(R,T)$ are as the following:

For $L_m = p^m$, 1st and 2nd INTERPRETATIONs, the trace of energy-momentum tensor and derivative relations:

$$\tau_0 = (-1 + 3w)\Omega_{m,0} \tag{17}$$

$$\tau_0^* = \frac{-2(1+w)[3(3 + F_{t,0}) + F_{tr,0}\mathfrak{R}_0^*]}{6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0}\tau_0 \tag{18}$$

$$\begin{aligned}\tau_0^{**} = {} & \frac{-2(1+w)}{[6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0]^2} \\ & \times \Big\{\{[-3(1+q_0)(3 + F_{t,0}) + 3(F_{tr,0}\mathfrak{R}_0^* + F_{tt,0}\tau_0^*) + (F_{trr,0}\mathfrak{R}_0^{*2} + F_{trt,0}\mathfrak{R}_0^*\tau_0^*) \\ & + F_{tr,0}\mathfrak{R}_0^{**}]\tau_0 + [3(3 + F_{t,0}) + F_{tr,0}\mathfrak{R}_0^*]\tau_0^*\} \times [6 + (3-w)F_{t,0} + 2(1+w)F_{tt,0}\tau_0] \\ & - [(3-w)(F_{tr,0}\mathfrak{R}_0^* + F_{tt,0}\tau_0^*) + 2(1+w)(F_{ttr,0}\mathfrak{R}_0^* + F_{ttt,0}\tau_0^*)\tau_0 \\ & + 2(1+w)F_{tt,0}\tau_0^*[3(3 + F_{t,0}) + F_{tr,0}\mathfrak{R}_0^*]\tau_0^*]\Big\}\end{aligned} \tag{19}$$

For $L_m = p^m$, 1st INTERPRETATION total effective mass and pressure parameters:

$$\Omega_{m,0}^{t.eff} = \frac{1}{F_{r,0}}\left[\Omega_{\mathrm{m},0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} - \frac{1}{6}(F_0 - F_{r,0}\mathfrak{R}_0) - (F_{rr,0}\mathfrak{R}_0^* + F_{rt,0}\tau_0^*)\right] \tag{20}$$

$$\begin{aligned}\Omega_{\mathrm{P},0}^{t.eff} = {} & \frac{1}{F_{r,0}}\left[w\Omega_{m,0} + \frac{1}{6}(F_0 - F_{r,0}\mathfrak{R}_0) + \frac{2}{3}(F_{rr,0}\mathfrak{R}_0^* + F_{rt,0}\tau_0^*)\right. \\ & \left. + \frac{1}{3}(F_{rrr,0}\mathfrak{R}_0^{*2} + F_{rtt,0}\tau_0^{*2} + 2F_{rrt,0}\mathfrak{R}_0^*\tau_0^* + F_{rr,0}\mathfrak{R}_0^{**} + F_{rt,0}\tau_0^{**})\right]\end{aligned} \tag{21}$$

For $L_m = p^m$, 2nd INTERPRETATION total effective mass and pressure parameters:

$$\Omega_{m,0}^{t.eff} = \Omega_{m,0} + \frac{1}{3}(1+w)\Omega_{m,0}F_{t,0} + (1 - F_{r,0})(1 - \Omega_{k,0}) - \frac{1}{6}(F_0 - F_{r,0}\mathfrak{R}_0) - (F_{rr,0}\mathfrak{R}_0^* + F_{rt,0}\tau_0^*) \tag{22}$$

$$\Omega_{\mathrm{P},0}^{t.eff} = w\Omega_{m,0} + \frac{1}{3}(1 - F_{r,0})(2q_0 - 1 + \Omega_{k,0}) + \frac{1}{6}(F_0 - F_{r,0}\Re_0) + \frac{2}{3}(F_{rr,0}\Re_0^* + F_{rt,0}\tau_0^*) + \frac{1}{3}(F_{rrr,0}\Re_0^{*2} + F_{rtt,0}\tau_0^{*2} + 2F_{rrt,0}\Re_0^*\tau_0^* + F_{rr,0}\Re_0^{**} + F_{rt,0}\tau_0^{**}) \quad (23)$$

Before proceeding the tests of the models, it would be better to briefly remind the outcomes based on the continuity equation in

$$(\kappa^2 + f_T)[\dot{\mu}^m + 3H(1 + w)\mu^m] = -\frac{1}{2}(1 - w)\dot{\mu}^m f_T - (1 + w)\mu^m \dot{f}_T \quad (24)$$

with the assumption of conservation of standard matter, the form of general function $f(R,G,T)$ is given by

$$w = -\frac{1}{3} \;\Rightarrow\; f(R,G,T) = K_1 \ln|T| + \phi(R,G) \quad (25.a)$$

$$w \neq \pm\frac{1}{3} \;\Rightarrow\; f(R,G,T) = K_2|T|^\gamma + \phi(R,G) \quad (25.b)$$

Here,

$$\gamma \equiv \frac{1 + 3w}{2(1 + w)} \quad (26)$$

constrained to $f(R,G,T) = \phi(R,G) + \psi(T)$ provided that the

$$2\kappa^2 + (3 - w)f_T + 2(1 + w)T\, f_{TT} \neq 0 \quad (27)$$

condition is valid. The relations argued can be expressed as (28-30) in the language of dimensionless variables,

$$6 + (3 - w)F_{t,0} + 2(1 + w)F_{tt,0}\tau_0 \neq 0 \quad (28)$$

$$F(r,t) = \Phi(r) + \Psi(t) \quad (29)$$

$$\Psi(t) = \begin{cases} K_1 \ln|t| & \text{for } w = -\frac{1}{3} \\ K_2\, |t|^\gamma & \text{for } w \in \left]-\frac{1}{3}, \frac{1}{3}\right[\cup \left]\frac{1}{3}, 1\right] \end{cases} \quad (30)$$

Here, the exponent γ was not arbitrary, but connected to w by the relation (26),

$$\gamma \equiv \frac{1 + 3w}{2(1 + w)} \quad (31)$$

and the changes of $\gamma = \gamma(w)$ and $w = w(\gamma)$ functions are given in Fig. 1 [19]. As can be seen clearly from these,

$$w \in \left]-\frac{1}{3}, \frac{1}{3}\right[\cup \left]\frac{1}{3}, 1\right] \Leftrightarrow \gamma \in \left]0, \frac{3}{4}\right[\cup \left]\frac{3}{4}, 1\right] \quad (32)$$

Although the dimensionless coefficient K is arbitrary, but must satisfy (28) which is the existence condition of τ_0^* and τ_0^{**} for a given value w or γ. According to this, if (28) is evaluated for (30), the following restrictions exist for K.

1) For $w = -\frac{1}{3}$, it is found that K is constrained in the form

$$K \neq 6\Omega_{m,0} \;\Rightarrow\; K_3 \neq 1.86 \quad (33)$$

when the derivatives of the function $\Psi(t) = K\ln(-t)$ are placed in (28), since in the present time $t = t_0$, $t_0 \equiv \tau_0 = -2\Omega_{m,0} < 0$.

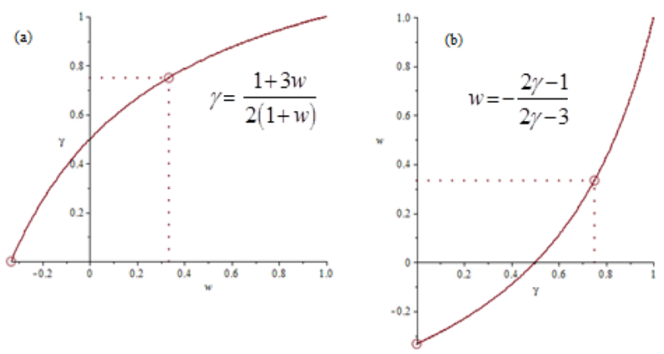

Fig. 1. For $L_m = p^m$, 1st and 2nd INTERPRETATIONs, in the case of conservation of standard matter, (a) the change of (γ, w) and (b) the change of (w, γ). In the Fig. 1.(a) the points $(w, \gamma) =$

$\left(-\frac{1}{3},0\right)$ and $\left(\frac{1}{3},\frac{3}{4}\right)$, and in the Fig. 1.(b) the points $(\gamma,w)=\left(0,-\frac{1}{3}\right)$ and $\left(\frac{3}{4},\frac{1}{4}\right)$ are illustrated by circles to show that they are excluded. For $-\frac{1}{3}<w<\frac{1}{3}$ or ($0<\gamma<\frac{3}{4}$) the $T<0$; and for $w=\frac{1}{3}$ or ($\gamma=\frac{3}{4}$) the $T=0$ and for $\frac{1}{3}<w\leq+1$ or ($\frac{3}{4}<\gamma\leq+1$) the $T>0$

2) Since in the interval of $-\frac{1}{3}<w<\frac{1}{3}$ $\left(\Rightarrow 0<\gamma<\frac{3}{4}\right)$, t_0 is $t_0\equiv\tau_0=(-1+3w)\Omega_{m,0}<0$ for $\Psi(t)=K\,(-t)^{\gamma}$ (28), with the use of (26 and 34),

$$w=-\frac{2\gamma-1}{2\gamma-3} \tag{34}$$

gives the constraint

$$K\neq 3(-\tau_0)^{1-\gamma}\Rightarrow K\neq\frac{3}{\gamma}\left(0.62\frac{4\gamma-3}{2\gamma-3}\right)^{1-\gamma}\Leftrightarrow K\neq\frac{6(1+w)}{1+3w}[0.31(1-3w)]^{\frac{2(1-w)}{1+w}} \tag{35}$$

after the needed adjustments.

3) For $\Psi(t)=K\,(t)^{\gamma}$ in the interval of $\frac{1}{3}<w\leq 1$ $\left(\Rightarrow\frac{3}{4}<\gamma\leq 1\right)$ in a similar way, the constraint

$$K\neq -3(\tau_0)^{1-\gamma}\Rightarrow K\neq-\frac{3}{\gamma}\left(-0.62\frac{4\gamma-3}{2\gamma-3}\right)^{1-\gamma}\Leftrightarrow K\neq-\frac{6(1+w)}{1+3w}[0.31(3w-1)]^{\frac{2(1-w)}{1+w}} \tag{36}$$

is found. In particular, for the value equals to 1 $(\Rightarrow\gamma=1)$, $\Psi(t)=K\,t$ with $K\neq-3$ since $\tau_0=2\Omega_{m,0}>0$. In the language of dimensional magnitudes, this indicates that for $w=1$, a functional form $\psi(T)=-\kappa^2T$ cannot exist. Nevertheless, a function in the form $\psi(T)=-\lambda\kappa^2T$ $(\lambda\neq-1)$ is possible, but it is also for only and only $w=1$. Fig. 2 shows all forbidden values of K in terms of γ and w for (30) [19].

3. DISCUSSIONS ABOUT THE MODELS AND CONCLUSIONS

Model 1: $\boldsymbol{F(r,t)=r+K_1 ln\,(-t)}$ **,** $\boldsymbol{w=-\frac{1}{3}}$ $(\boldsymbol{L_m=p^m\ and\ conservation\ valids})$

We list the results of the calculations regarding the 1st INTERPRETATION of the sizes included in the equations (17 - 23) below, in terms of K_1, by taking $\Omega_{k,0}$ as the free parameter (In a 10^{-12} sensitivity calculation, some numbers were rounded up to ten thousandth digits). Fig. 2 shows the energy conditions:

$$\Re_0=10.86-6\Omega_{k,0}$$

$$\Re_0^{*}=5.82+12\Omega_{k,0}$$

$$\Re_0^{**}=-26.28\Omega_{k,0}-0.2634$$

$$\tau_0=-0.62$$

$$\tau_0^{*}=\frac{0.8267(3-1.6129K_3)}{2-1.0753K_3}$$

$$\tau_0^{**}=\frac{0.3333(-32.5872+35.0400K_3-9.4194K_3^2)}{(2-1.0753K_3)^2}$$

$$F_0=10.86-6\Omega_k-0.4780K_3$$

$$F_{r,0}=1$$

$$F_{rr,0}=0$$

$$F_{t,0}=-1.6129K_3$$

$$F_{tt,0}=-2.6015K_3$$

$$F_{ttt,0}=-8.3918K_3$$

$$\Omega_{m,0}^{t.eff}=0.3100-0.0314K_3$$

$$\Omega_{P,0}^{t.eff}=-0.1033-0.0797K_3$$

$$NEC: 0.2067-0.1111K_3\geq 0$$

$$WEC: 0.3100-0.0314K_3\geq 0$$

$$SEC: 1\times10^{-11}-0.2705K_3<0$$

$$DEC: 0.4133+0.0482K_3\geq 0$$

Considering the condition (33), we find the coupling constant K_1 , which is the common solution of the above inequalities, which is the expression of the *1st set of conditions*, restricted by the interval $3.69745393711 \times 10^{-11} < K_1 < 1.86$. The following graphical representation also confirms this [19]. In this $F(r,t)$ model, the value of $w_{DE,0}$, which is the measure of superacceleration, is $w_{DE,0} = -(9.54244679439 \times 10^{-11}/K_1) + 2.53423955351$ and requires a fine-tuning such as $-1.2 \leq w_{DE} \leq -0.8$ and $0 < K_1 < 2.69999999998 \times 10^{-11}$ for $2.555392244570 \times 10^{-11} < K_1 < 2.86195596964 \times 10^{-11}$ and $w_{DE,0} < -1$. Considering that current observational data do not have such sensitivity, it is important to notice that this range is not meaningful. Related to this model, let's note as a final point that: the results are independent of the parameter in calculations made without attributing a value to $\Omega_{k,0}$. On the other hand, since the model we have dealt with is of type $f(R,G,T) = R + F(T)$, the 1st and 2nd INTERPRETATIONs are equivalent to each other.

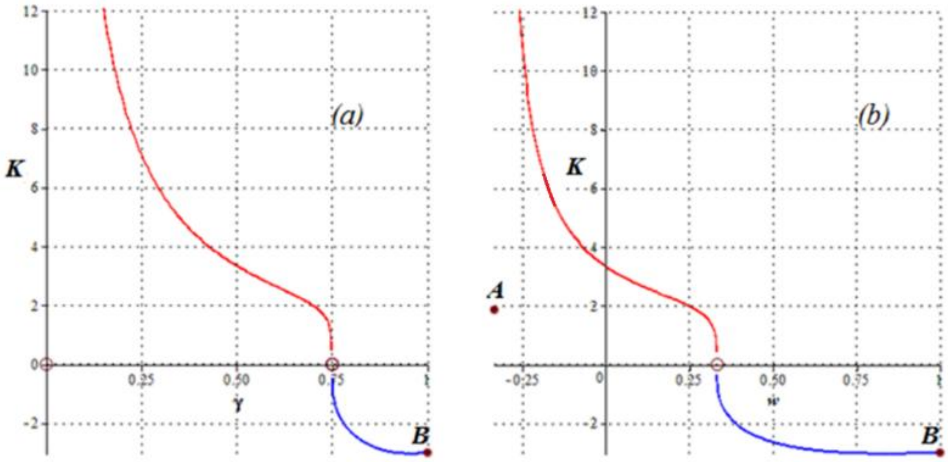

Fig. 2. For the function in (30): all forbidden values of K, 2.(a) according to γ and 2.(b) according to w. Red curve is drawn using (35); the blue curve is drawn using (36). In 2.(b), the coordinates of points A and B are (–0.33, 1.86) and (1, –3), respectively. The small empty circles show the γ and w values for which the function is not defined. Let us emphasize that $\Psi(t) = K \ln(-t)$ and $K \neq 1.86$ are for $w = -\frac{1}{3}$; while $\Psi(t) = K\,t$ and $K \neq -3$ are for $w = 1$

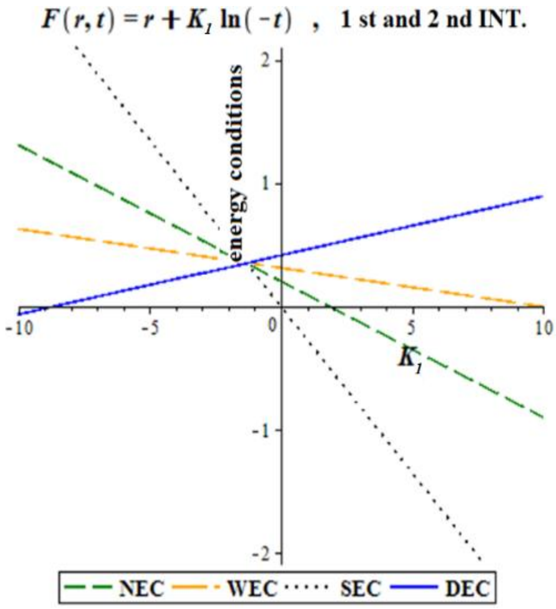

Fig. 3. For model $F(r,t) = r + K_1 \ln(-t)$, the energy conditions in the case of $w = -\frac{1}{3}$ where the standard matter is conserved.

Model 2: $F(r,t) = r + K_2(-t)^{\gamma}$, $-\frac{1}{3} < w < \frac{1}{3} \Leftrightarrow 0 < \gamma < \frac{3}{4}$ $(L_m = p^m \textit{ and conservation valids})$

We first study the testing of this model under the 1st INTERPRETATION, again considering the $\Omega_{k,0}$ parameter as free. Because their expressions are overly complex, we prefer not to write the relevant equations and show the results only with graphics. We show the results in Figs. 3 and 4 by taking the K_2 parameter in the $K_2 \in [-10, +10]$ interval and the γ parameter in the $\gamma \in]0, 0.75[$ interval, respectively, with the steps $\Delta K_2 = 0.2$ and $\Delta\gamma = 0.01$, and taking into account the relation between γ and w in (26), and also adding the restrictions to the *1st and 2nd sets of conditions* in (35). In Fig. 4, for some selected special values of K_2 in the interval of $[-10, +10]$: the changes of NEC, WEC, SEC and DEC according to the exponent γ and the change of $w_{KE,0}$ are shown [19]. The other, it is independent of K in accordance with the $w_{DE,0} = (\gamma - 1.5)(4\gamma - 3)/2(\gamma^2 - 3\gamma + 0.25)$ result. It is understood from the aforementioned graphs that there are (K_2, γ) pairs that satisfy the *1st set of conditions* and none of them contain super acceleration. We create Fig. 5 to both clarify the pairs in question and to see if the *2nd set of conditions* is satisfied. From here, it can be seen that the *1st set of conditions* restricts the widest interval of definition, which is $0 < \gamma < 3/4 = 0.75$ for γ, to $0 < \gamma < 0.56$ for each $K_2 < 0$; and the *2nd set of conditions* restricts $0 < \gamma \leq 0.25$. These intervals correspond to the $-1/3 < w < +0.064$ and $-1/3 < w < -1/5$, respectively, in accordance with (34). Let us note that the results obtained for the model in question are also identical for the 2nd INTERPRETATION and independent of the value of $\Omega_{k,0}$.

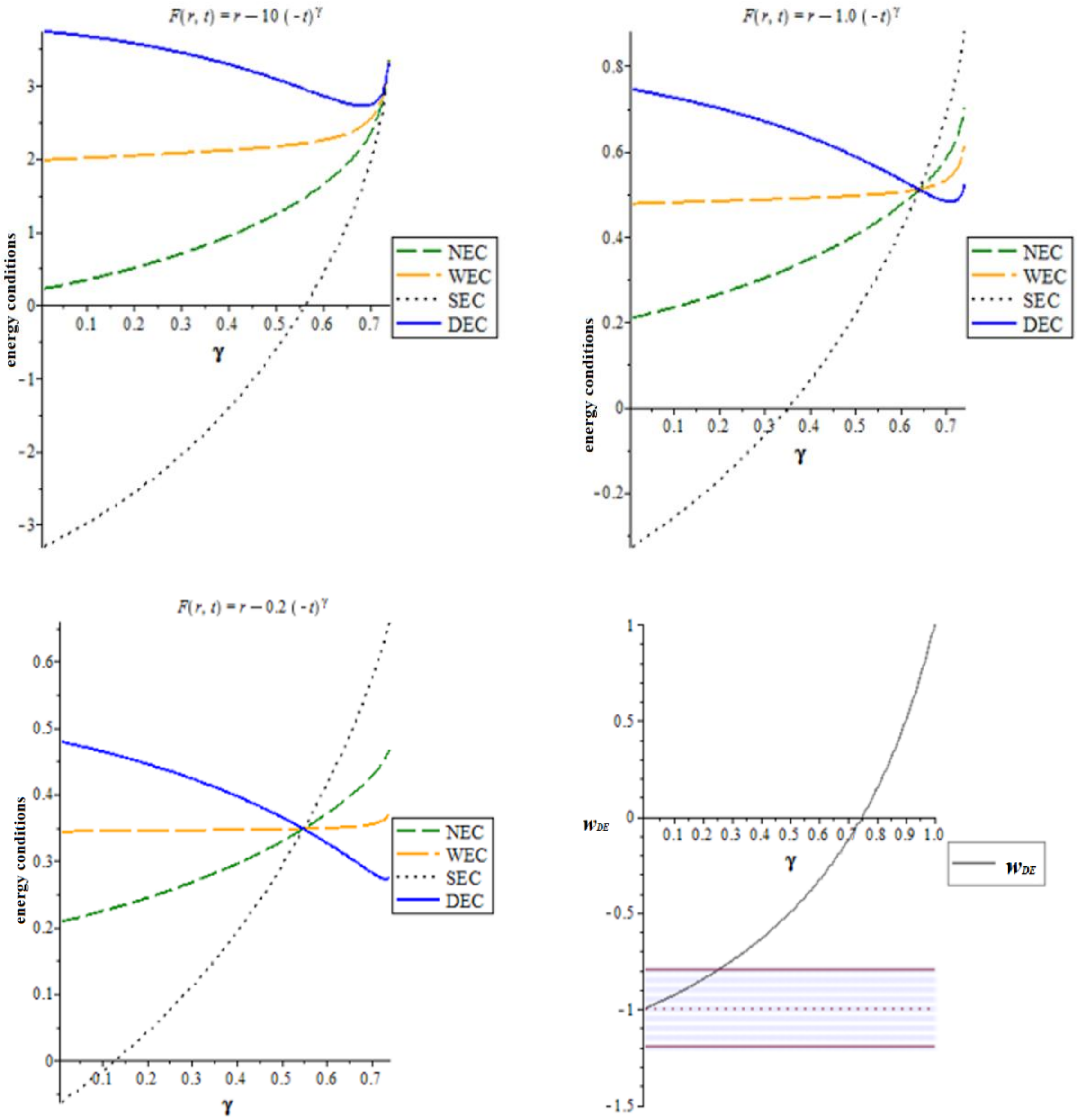

Fig. 4. Variations of NEC, WEC, SEC and DEC for $F(r,t) = r + K_2(-t)^\gamma$ with respect to γ for some selected specific K_2 values. In the right bottom panel, there is the change of $w_{DE,0}$

regardless of K_2. As can be seen, for the interval $0 < \gamma < \frac{3}{4}$. It is always $w_{DE,0} > -1$, that is, there is no super acceleration

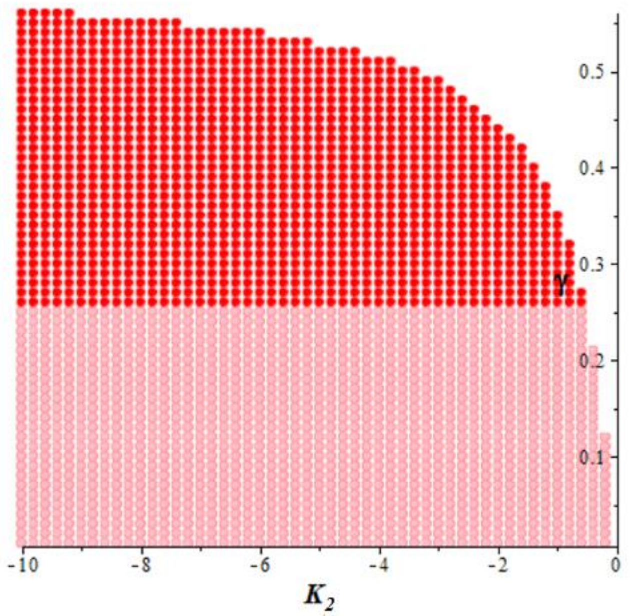

Fig. 5. The pairs (K_2, γ) that provide the *1st set of conditions* (pink-looking region+only red zone) and the *2nd set of conditions* (pink-looking region). The ratio of the number of models providing the *2nd condition set* in the interval $(-10 \le K_2 < 0) \times (0 < \gamma < 0.56)$ to that of the *1st condition set* is: 1233/2444 = 50.5%

4. CONCLUSION

For the first model, considering the condition (33), we find the coupling constant K_1, which is the common solution of the above inequalities, which is the expression of the *1st set of conditions*, restricted by the interval $3.69745393711 \times 10^{-11} < K_1 < 1.86$. The graphical representation in Fig. 3 also confirms this. In this $F(r,t)$ model, the value of $w_{DE,0}$, which is the measure of superacceleration, is $w_{DE,0} = -(9.54244679439 \times 10^{-11}/K_1) + 2.53423955351$ and requires a fine-tuning such as $-1.2 \le w_{DE} \le -0.8$ and $0 < K_1 < 2.69999999998 \times 10^{-11}$ for $2.555392244570 \times 10^{-11} < K_1 < 2.86195596964 \times 10^{-11}$ and $w_{DE,0} < -1$. Considering that current observational data do not have such sensitivity, it is important not to find these value ranges meaningful. Related to this model, let's note as a final point that: the results are independent of the parameter in calculations made without attributing a value to $\Omega_{k,0}$. On the other hand, since the model we have dealt with is of type $f(R,G,T) = R + F(T)$, the 1st and 2nd INTERPRETATIONs are equivalent to each other.

Related to the second model, variations of NEC, WEC, SEC and DEC with respect to γ for some selected specific K_2 values are obtained in the Fig. 4. As can be seen, for the interval $0 < \gamma < 3/4$. It is always $w_{DE,0} > -1$, that is, there is no super acceleration. The pairs (K_2, γ) that provide the *1st set of conditions* and the *2nd set of conditions* are identified in the Fig. 5. Besides the ratio of the number of models providing the *2nd condition set* in the interval $(-10 \le K_2 < 0) \times (0 < \gamma < 0.56)$ to that of the *1st condition set* is: 1233/2444 = 50.5%.

As a last remark, let us repeat that, in the General Discussions part of this work, the equations for $f(R,T)$ models are obtained for conservative and non-conservative cases and for two interpretations and also for both $L_m = p^m$ and $L_m = -\mu^m$ cases. Besides w and w_{DE} are obtained for them. The change of parameters of the functions are obtained by the change of the others. Moreover, the satisfaction of the energy conditions are also restricted to be viable by simulations and by that way the range of values for the parameters of the functions are found.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Copeland EJ, Sami M, Tsujikawa S. Dynamics of Dark Energy, Int. J. Mod. Phys. 2006;D15:1753-1936. DOI: arXiv:hep-th/0603057
2. Clegg, B., Dark Matter and Dark Energy: The Hidden 95% of the Universe, Icon Books; 2019
3. Lapuente PR. Dark energy: Observational and theoretical approaches, Cambridge University Press; 2010.
4. Jennings E. Simulations of dark energy cosmologies, Springer Science & Business Media; 2012.
5. Amendola L, Tsujikawa S. Dark energy: Theory and Observations, Cambridge University Press; 2010.
6. Wang Y. Dark Energy, John Wiley & Sons; 2009.
7. Ade PAR, et al. Planck 2013 Results. I. Overview of Products and Scientific Results, Astron. Astrophys. 2014;571:A1. DOI: arXiv:1303.5062 [astro-ph.CO]
8. Riess AG, et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Astron. J. 1998;116:1009-1038. DOI: e-Print: astro-ph/9805201 [astro-ph]
9. Perlmutter S, et al. Measurements of Omega and Lambda from 42 High-Redshift Supernovae, Astrophys. J. 1999;517:565-586. DOI: e-Print: astro-ph/9812133 [astro-ph]
10. Durrer R. What do we really know about dark energy?, Phil. Trans. Roy. Soc. Lond. A. 2011;369:5102-5114, J. Cosmol. 2011;15:6065. DOI: e-Print: 1103.5331 [astro-ph.CO]
11. Spergel DN, et al. Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology, Astrophys. J. Suppl. 2007;170:377. DOI: e-Print: astro-ph/0603449 [astro-ph]
12. Clifton T, Ferreira PG, Padilla A, Skordis C. Modified Gravity and Cosmology, Physics Reports. 2012;513(1–3): 1-189.
13. Kroupa P, Pawlowski M, Milgrom M. The Failures of the Standard Model of Cosmology Require a New Paradigm, Int. J. Mod. Phys. D. 2012;21:1230003. DOI: e-Print: 1301.3907 [astro-ph.CO]
14. Martinelli M, et al. Constraining Modified Gravitational Theories by Weak Lensing with Euclid, Phys. Rev. D. 2011;83:023012.
15. Easson DA. Modified gravitational theories and cosmic acceleration, International Journal of Modern Physics A. 2004;19(31):5343-5350.
16. Bamba K, Nojiri S, Odintsov SD. The Future of the Universe in Modified Gravitational Theories: Approaching a Finite-time Future Singularity, JCAP10. 2008;045.
17. Gudekli E, Myrzakul A, Myrzakulov R. Teleparallelism by Inhomogeneous Dark Fluid, Astrophysics and Space Science. 2015;359:N2,64.
18. Zubair M, Javaid H, Azmat H, Güdekli E. Relativistic Stellar Model in $f(R,T)$ Gravity Using Karmarkar Condition, New Astronomy. 2021;88:101610.
19. Can D. Infrared Corrections to General Relativity as an Alternative to Dark Matter and Dark Energy, Ph.D. Thesis, Istanbul University Institute of Science; 2021.

Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle4.com/review-history/73259