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Numerical Solution of First Kind Fredholm Integral Equations Using Wavelet Collocation Method

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

We developed an algorithm based on combination of regularization and wavelet collocation method to solve Fredholm integral equations of the first kind. As first kind Fredholm integral equations are often ill-posed problems, regularization method is implemented to convert it into an approximate well posed Fredholm integral equation of the second kind whose solution converges to the solution of the original problem. Then wavelet collocation method is applied to obtain the numerical solution of the resulting problem. We have applied proposed method using Legendre and Chebyshev wavelets to some examples and compared their efficiency.

Keywords: Chebyshev wavelet; Legendre wavelet; collocation method; ill-posed problem; regularization method.

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1 Introduction

Fredholm integral equation of the first kind is an important type of integral equation which arises in different scientific applications. It appears in many different fields like Mathematics (backward heat equation, differentiation), Physics (geophysics, atomic physics, spectroscopy, gravitational problem), Image processing (medical imaging, radiography) and physical problems (angular variation of scattered light, measurement of spectral distribution). Inverse problem of Laplace transform and diffuse optical tomography can be easily expressed in the form of first kind Fredholm integral equation.

The Fredholm integral equation of the first kind is given as

$$v(x) = \int_0^1 K(x,t)F(u(t))dt, \qquad x \in [0,1), \tag{1.1}$$

where $F(u(t))$ is a function of $u(t)$ and $K(x,t)$ represents kernel of equation. Equation (1.1) is called linear or nonlinear depending upon whether $F(u(t))$ is linear or nonlinear function of $u(t)$. Here, the problem is to determine the function $u(x)$ for given values of $K(x,t)$ and $v(x)$ which is an inverse problem. It is an ill-posed problem [1]. Because of its ill-posed nature, regularization method is required to solve this problem.

Regularization method converts an ill-posed problem into a well posed problem. There are different techniques used for regularizing first kind Fredholm integral equation. In this paper, we have used regularization method to convert first kind integral equation into second kind integral equation. As second kind Fredholm integral equation is a well posed problem, we can apply any suitable method to find solution to this problem. The regularization method is combined with different techniques like homotopy perturbation method [2], direct method, successive approximation and Adomian decomposition method [3] and mean value method [4] to solve first kind Fredholm integral equations. Adomian decomposition method has also been applied to solve vector born and smoking model [5, 6, 7]. For more details about these methods see [2].

Wavelet based methods have been successfully employed to solve integral equations. Haar wavelet collocation method [8] and Wavelet Galerkin [9] have been developed to solve Fredholm integral equations of the first kind. Maleknejad et.al. [10] applied Legendre wavelet Galerkin discretization and CG method to solve Fredholm integral equations of the first kind. Finite difference method has been applied to solve non-linear Volterra integro-differential equation by Cakir et.al.[11]. Numerical solution of Volterra-Fredholm integral equation systems have been obtained by Bernstein multi-scaling polynomials [12]. Also, solution to Fredholm and Volterra integral equations of the second kind has been obtained using Legendre wavelet [13] and Chebyshev wavelet [14]. In this paper, we proposed a technique wavelet collocation method combined with regularization method.

The paper is organized as follows. Section 2 provides a brief introduction to wavelets and its approximation properties. Section 3 describes the proposed numerical method. In section 4, the proposed method is applied to some numerical problems using Legendre and Chebyshev wavelets and presents their results. The conclusion is drawn in section 5.

2 Wavelets

We have used Legendre and Chebyshev wavelets for obtaining numerical solution of first kind Fredholm integral equations. In this section we will give introduction to these wavelets.

2.1 Legendre wavelets

Let $P_m(x)$ be Legendre polynomials of order $m \geq 0$, then the Legendre wavelets $\psi_{n,m}(x)$ on interval [0,1) are defined by

$$\psi_{n,m}(x) = \begin{cases} \sqrt{(m+\frac{1}{2})}2^{k/2}P_m(2^k x - \hat{n}), & \text{for } \frac{\hat{n}-1}{2^k} \leq x < \frac{\hat{n}+1}{2^k}, \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

where $\hat{n} = 2n-1,\ n = 1, 2, \ldots, 2^{k-1}; k \geq 1$. The polynomials $P_0(x)$ and $P_1(x)$ are given as

$$P_0(x) = 1$$

$$P_1(x) = x$$

and all higher degree Legendre polynomials are obtained using recursive formula

$$P_{m+1}(x) = \left(\frac{2m+1}{m+1}\right) xP_m(x) - \left(\frac{m}{m+1}\right) P_{m-1}(x), \quad \text{for} \quad m = 1, 2, 3, \ldots.$$

Legendre wavelets form a complete orthonormal basis for $L^2[0, 1)$ [15].

2.2 Chebyshev wavelet

Let $T_m(x)$ be Chebyshev polynomials of the first kind of degree m defined by

$$\begin{aligned} T_0(x) &= 1, \\ T_1(x) &= x, \\ T_{m+1}(x) &= 2xT_m(x) - T_{m-1}(x), \quad m = 1, 2, ... \end{aligned}$$

Chebyshev polynomials $T_m(x)$ are orthogonal with respect to the weight function $w(x) = 1/\sqrt{(1-x^2)}$, on the interval [-1,1]. Chebyshev wavelets are defined on [0,1) as

$$\psi_{n,m}(x) = \begin{cases} \frac{\alpha_m 2^{k/2}}{\sqrt{(\pi)}} T_m(2^k x - 2n + 1), & \text{for } \frac{n-1}{2^{k-1}} \leq x < \frac{n}{2^{k-1}}, \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

where

$$\alpha_m(x) = \begin{cases} 1, & m = 0 \\ \sqrt{2}, & m > 0 \end{cases}, \tag{2.3}$$

$n = 1, 2, \ldots, 2^{k-1},\ k = 1, 2, 3, \ldots.$ and $m = 0, 1, 2, \ldots, M-1$ where M is a positive integer. The Chebyshev wavelets $\psi_{n,m}(x)$ form an orthonormal basis with weight function $w_n(x) = w(2^k x - 2n + 1)$ on $L^2[0, 1)$ [14].

2.3 Approximation of function

Any square integrable function $u(x)$ defined over $[0, 1)$ can be written as sum of series of wavelets as

$$u(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m} \psi_{n,m}(x), \tag{2.4}$$

where $\psi_{n,m}(x)$ represents wavelets and $c_{n,m}$ are wavelet coefficients given by

$$c_{n,m} = \langle u(x), \psi_{n,m}(x) \rangle = \int_0^1 u(x)\psi_{n,m}(x)dx. \tag{2.5}$$

After truncating the series at $m = 0, 1, 2, \ldots, M-1$ and $n = 1, 2, \ldots, 2^{k-1}$, (2.4) can be written as sum of series of $2^{k-1}M$ terms.

$$u(x) \simeq \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x), \tag{2.6}$$

Writing it in matrix notation,

$$u(x) = C\Psi^T(x),$$

where C and $\Psi(x)$ are $1 \times 2^{k-1}M$ matrices and are given by

$$C = [c_{1,0}, c_{1,1}, \ldots, c_{1,M-1}, c_{2,0}, \ldots, c_{2,M-1}, \ldots, c_{2^{k-1},0} \ldots c_{2^{k-1},M-1}], \tag{2.7}$$

$$\begin{aligned}\Psi(x) \quad = \quad & [\psi_{1,0}(x), \psi_{1,1}(x), \ldots, \psi_{1,M-1}(x), \psi_{2,0}(x), \ldots, \psi_{2,M-1}(x), \ldots \\ & \ldots, \psi_{2^{k-1},0}(x), \ldots, \psi_{2^{k-1},M-1}(x)].\end{aligned} \tag{2.8}$$

The convergence for Legendre wavelet series is given by following theorem.

Theorem 2.1. [16] A function $u(x) \in L^2[0,1)$ with bounded second derivative, say $|u''(x)| \leq N$ can be expanded as an infinite sum of Legendre wavelets, and the series converge uniformly to $u(x)$, i.e.,

$$u(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x)$$

and the value of wavelet coefficients $c_{n,m}$ is given by

$$|c_{n,m}| < \frac{\sqrt{12}M}{(2n)^{5/2}(2m-3)^2}.$$

The convergence for Chebyshev wavelet series is given by following theorem.

Theorem 2.2. [17] A function $u(x) \in L^2[0,1)$ with bounded second derivative, say $|u''(x)| \leq N$ can be expanded as an infinite sum of Chebyshev wavelets, and the series converges uniformly to $u(x)$, i.e.,

$$u(x) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x)$$

and the value of wavelet coefficients $c_{n,m}$ is given by

$$|c_{n,m}| < \frac{\sqrt{2\pi}N}{(2n)^{5/2}(m^2-1)}.$$

3 Numerical Method

First, we convert ill-posed first kind Fredholm integral equations into well posed second kind Fredholm integral equations using regularization method. This regularization method has been used by many authors and is found to be very reliable [4, 2, 18].

Consider Fredholm integral equation of the first kind of form

$$v(x) = \int_0^1 K(x,t)u(t)dt, \quad x \in [0,1). \tag{3.1}$$

where $u(x) \in L^2[0,1)$. We will transform (3.1) into a well posed second kind Fredholm integral equation using regularization method.

$$\alpha u_\alpha(x) = v(x) - \int_0^1 K(x,t)u_\alpha(t)dt, \tag{3.2}$$

where $\alpha > 0$ is known as regularization parameter and $u_\alpha(x)$ converges to $u(x)$ as $\alpha \to 0$ [2], i.e.,

$$u(x) = \lim_{\alpha \to 0} u_\alpha(x). \tag{3.3}$$

Now we will apply wavelet collocation method to handle the resulting well posed problem. To employ this method, expand $u_\alpha(x)$ as sum of series of wavelets as

$$u_\alpha(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x). \tag{3.4}$$

Putting these values in (3.2), we get

$$\begin{aligned} \alpha \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x) &= v(x) - \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} \int_0^1 K(x,t)\psi_{n,m}(t)dt \\ \alpha \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x) &= v(x) - \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} R_{n,m}(x), \end{aligned} \tag{3.5}$$

where $R_{n,m}(x) = \int_0^1 K(x,t)\psi_{n,m}(t)dt$.
Define collocation points as

$$x_i = \frac{2i-1}{2p}, \quad i = 1, 2, \ldots, q,$$

where $q = 2^{k-1}M$. Satisfying (3.5) at all collocation points x_i, we get the following linear system of equations,

$$\alpha \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x_i) = v(x_i) - \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m} R_{n,m}(x_i), \quad i = 1, 2, \ldots, q.$$

Writing this system in matrix form, we have

$$\begin{aligned} \alpha C\Psi &= V - CR \\ C(\alpha\Psi + R) &= V \\ C &= V(\alpha\Psi + R)^{-1}. \end{aligned} \tag{3.6}$$

The function V, Ψ and R are given by

$$V = [v(x_1), v(x_2), ..., v(x_q)],$$

$$\Psi = \begin{bmatrix} \psi_{1,0}(x_1) & \psi_{1,0}(x_2) & . & . & . & . & . & \psi_{1,0}(x_q) \\ \psi_{1,1}(x_1) & \psi_{1,1}(x_2) & . & . & . & . & . & \psi_{1,1}(x_q) \\ . & . & . & . & . & . & . & . \\ \psi_{1,M-1}(x_1) & \psi_{1,M-1}(x_2) & . & . & . & . & . & \psi_{1,M-1}(x_q) \\ \psi_{2,0}(x_1) & \psi_{2,0}(x_2) & . & . & . & . & . & \psi_{2,0}(x_q) \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \psi_{2^{k-1},M-1}(x_1) & \psi_{2^{k-1},M-1}(x_2) & . & . & . & . & . & \psi_{2^{k-1},M-1}(x_q) \end{bmatrix},$$

$$R = \begin{bmatrix} R_{1,0}(x_1) & R_{1,0}(x_2) & . & . & . & . & . & R_{1,0}(x_q) \\ R_{1,1}(x_1) & R_{1,1}(x_2) & . & . & . & . & . & R_{1,1}(x_q) \\ . & . & . & . & . & . & . & . \\ R_{1,M-1}(x_1) & R_{1,M-1}(x_2) & . & . & . & . & . & R_{1,M-1}(x_q) \\ R_{2,0}(x_1) & R_{2,0}(x_2) & . & . & . & . & . & R_{2,0}(x_q) \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ R_{2^{k-1},M-1}(x_1) & R_{2^{k-1},M-1}(x_2) & . & . & . & . & . & R_{2^{k-1},M-1}(x_q) \end{bmatrix}.$$

Eq.(3.6) is solved to find unknown coefficients $C = \{c_{n,m}\}$ and $u_\alpha(x)$ is calculated using (3.4). Then $u(x)$ is obtained from $u_\alpha(x)$ using (3.3).

The value of α is chosen in such a way that numerical solution converges to exact solution. To compare efficiency of wavelets we use root mean square error (rms)

$$\text{rms} = \frac{1}{N}\sum_{i=1}^{N} |u(x_i) - u_{ex}(x_i)|,$$

where $u(x_i)$ and $u_{ex}(x_i)$ are numerical and exact solution and N is the number of points at which data is measured.

Also, nonlinear Fredholm integral equation of the first kind can be solved by same method by converting it into linear Fredholm integral equation of the first kind by putting

$$F(u(x)) = v(x).$$

4 Numerical Examples

In this section, we presents some examples to check efficiency of the proposed method. The numerical results obtained using Legendre and Chebyshev wavelets are shown in tables and figures. All examples are solved using $k = 2,\ M = 3$, i.e., collocation points $q = 6$. All computations are done with help of Matlab.

Example 4.1. Solve the following first kind Fredholm integral equation:

$$\int_0^1 e^{t\sin x} u(t)dt = -\frac{e^{\sin x}(0.54\sin x + 0.84) - \sin x}{\cos x^2 - 2}, \quad 0 \le x \le 1.$$

The exact solution is $u(x) = \cos x$.

Chebyshev wavelet gives accurate results at $\alpha = 10^{-4}$ with rms value 3.31917747e-03. Legendre wavelet gives good results for $\alpha = 10^{-4}$ and rms value obtained is 3.95338299e-03. The results obtained are shown in Table 1 and Fig. 1 .

Example 4.2. Consider the following first kind Fredholm integral equation:

$$\int_0^1 e^t(\sin(x - t + 1) + 1)u(t)dt = 1 + \cos(x) - \cos(x + 1), \quad 0 \le x \le 1.$$

The exact solution is $u(x) = e^{-x}$.

Chebyshev wavelets gives rms=4.987735534706514e-03 at $\alpha = 10^{-8}$ and Legendre wavelet gives rms=6.07245313e-03 for $\alpha = 10^{-8}$. The results obtained are compared in Table 2 and Fig. 2.

Table 1. Exact and numerical solution for Example 4.1

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\|u_{ex}(x) - u(x)\|$	Error using Legendre wavelet $\|u_{ex}(x) - u(x)\|$
0	1.00000000	5.71557321e-03	1.48847206e-03
0.1	0.99500416	1.39596756e-03	2.02011478e-03
0.2	0.98006657	1.53444359e-03	2.53185302e-03
0.3	0.95533648	2.92640884e-03	1.95994098e-04
0.4	0.92106099	2.53283330e-03	4.74036713e-03
0.5	0.87758256	4.09080798e-03	6.58689337e-03
0.6	0.82533561	2.53117784e-03	2.92061636e-03
0.7	0.76484218	4.76224551e-03	6.56532489e-03
0.8	0.69670670	3.20682536e-03	4.95166254e-03
0.9	0.62160996	1.45429542e-03	1.23958349e-03

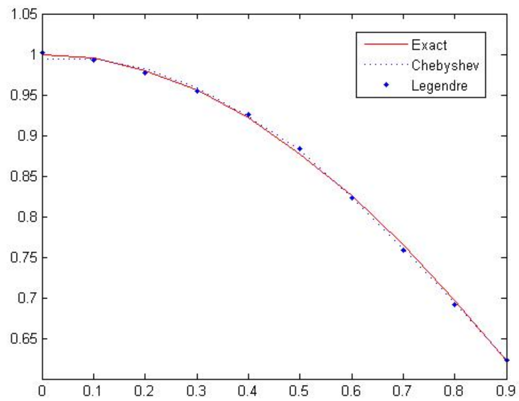

Fig. 1. Plot of exact and numerical solution for Example 4.1

Table 2. Exact and numerical solution for Example 4.2

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\|u_{ex}(x) - u(x)\|$	Error using Legendre wavelet $\|u_{ex}(x) - u(x)\|$
0	1.00000000	1.26155666e-02	1.51007686e-02
0.1	0.90483741	2.15261123e-03	1.71091928e-03
0.2	0.81873075	3.49454337e-03	5.27043699e-03
0.3	0.74081822	5.18768159e-03	6.70508465e-03
0.4	0.67032004	3.70657822e-03	3.37279851e-03
0.5	0.60653065	1.56230523e-03	6.14034096e-03
0.6	0.54881163	2.09369718e-03	3.46187840e-03
0.7	0.49658530	3.85879937e-03	1.74263893e-03
0.8	0.44932896	3.21030266e-03	4.59923853e-04
0.9	0.40656965	3.24750286e-04	8.59224167e-04

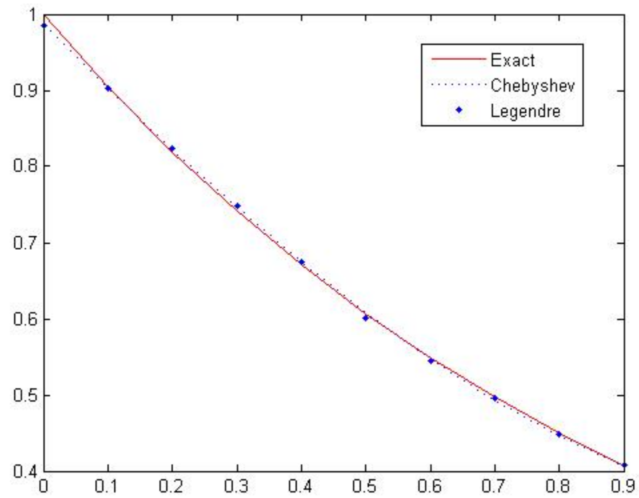

Fig. 2. Plot of exact and numerical solution for Example 4.2

Example 4.3. Consider Fredholm integral equation of the first kind

$$\int_1^3 (E+E')^{-1}\rho(E')dE' = E^{-1}ln\left(\frac{1+E/a}{1+E/b}\right), \quad 1 \le E \le 2,$$

with exact solution $p(E) = E^{-1}$.

This problem arises in field of electron atom scattering problem. Here the value of rms=3.55584416e-03 is obtained using Chebyshev wavelet at $\alpha = 10^{-10}$. Legendre wavelet gives rms= 3.66677100e-02 at $\alpha = 10^{-6}$. The results obtained are shown in Table 3 and Fig. 3.

Table 3. Numerical results for Example 4.3

E	Exact solution $\rho_{ex}(E)$	Error using Chebyshev wavelet $\lvert\rho_{ex}(E)-\rho(E)\rvert$	Error using Legendre wavelet $\lvert\rho_{ex}(E)-\rho(E)\rvert$
0	1.00000000	7.67691060e-03	5.13346488e-02
0.1	0.83333333	3.97340715e-03	1.46594466e-02
0.2	0.71428571	1.40485767e-03	3.30253721e-02
0.3	0.62500000	5.95456224e-03	2.16202708e-02
0.4	0.55555555	2.44928366e-04	9.63522261e-03
0.5	0.50000000	4.09051930e-05	3.44779193e-02
0.6	0.45454545	1.02978267e-03	5.22655310e-02
0.7	0.41666666	2.47754457e-03	4.44633333e-02
0.8	0.38461538	2.63593915e-03	9.32307457e-03
0.9	0.35714285	2.56215158e-04	5.44039965e-02

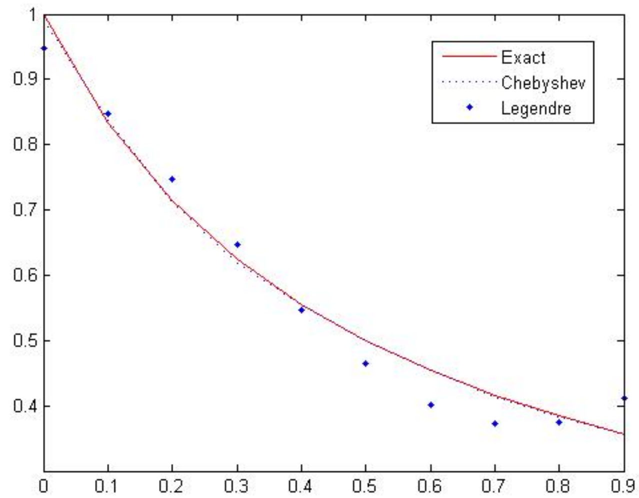

Fig. 3. Plot of exact and numerical solution for Example 4.3

Example 4.4. Consider the following integral equation

$$\int_0^1 \sin(xt)u(t)dt = \frac{\sin(x) - x\cos(x)}{x^2}, \quad 0 \leq x \leq 1,$$

with exact solution $u_{ex} = x$.

For Chebyshev wavelet the minimum value of rms is obtained for regularization parameter $\alpha = 10^{-9}$ and its value is rms=6.18630084e-07 whereas Legendre wavelet gives rms=7.2886464e-07 at $\alpha = 10^{-8}$.The exact solution and absolute errors are shown in Table 4 and Fig. 4.

Table 4. Numerical results for Example 4.4

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\lvert u_{ex}(x) - u(x)\rvert$	Error using Legendre wavelet $\lvert u_{ex}(x) - u(x)\rvert$
0	0	1.57931568e-06	5.35284295e-07
0.1	0.1	1.22161464e-07	5.84695099e-07
0.2	0.2	5.58852303e-07	1.13921518e-06
0.3	0.3	4.63725617e-07	1.12827595e-06
0.4	0.4	4.07541520e-07	5.51877424e-07
0.5	0.5	7.11529881e-07	1.09102228e-06
0.6	0.6	3.29155136e-07	2.13893066e-07
0.7	0.7	7.61371329e-08	3.29615161e-07
0.8	0.8	4.75241279e-08	5.39502397e-07
0.9	0.9	4.18286467e-08	4.15768641e-07

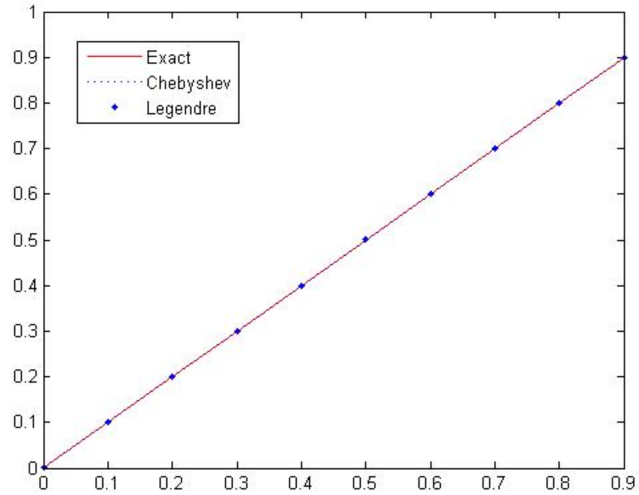

Fig. 4. Plot of exact and numerical solution for Example 4.4

Example 4.5. Solve the following integral equation

$$\int_0^1 e^{xt}u(t)dt = \frac{e^{x+1}-1}{x+1}, \quad 0 \le x \le 1.$$

Exact solution to this problem is $u_{ex} = x$.

The optimal value of rms obtained 1.05867302e-03 is at $\alpha = 10^{-4}$ using Chebyshev wavelet and for Legendre wavelets we get rms =9.00670259e-03 at $\alpha = 10^{-6}$.

Table 5. Numerical results for Example 4.5

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\|u_{ex}(x) - u(x)\|$	Error using Legendre wavelet $\|u_{ex}(x) - u(x)\|$
0	1.00000000	1.35271584e-03	1.22334929e-02
0.1	1.10517091	4.05806339e-04	7.62623195e-03
0.2	1.22140275	3.89218922e-04	4.58136562e-03
0.3	1.34985880	2.39190779e-04	4.26218127e-03
0.4	1.49182469	1.93791446e-04	7.95431022e-03
0.5	1.64872127	2.92369155e-03	1.44208237e-02
0.6	1.82211880	2.37555122e-04	8.71257258e-03
0.7	2.01375270	4.44572538e-04	5.18342313e-03
0.8	2.22554092	3.84702754e-04	5.75131196e-03
0.9	2.45960311	1.30623055e-04	1.25358867e-02

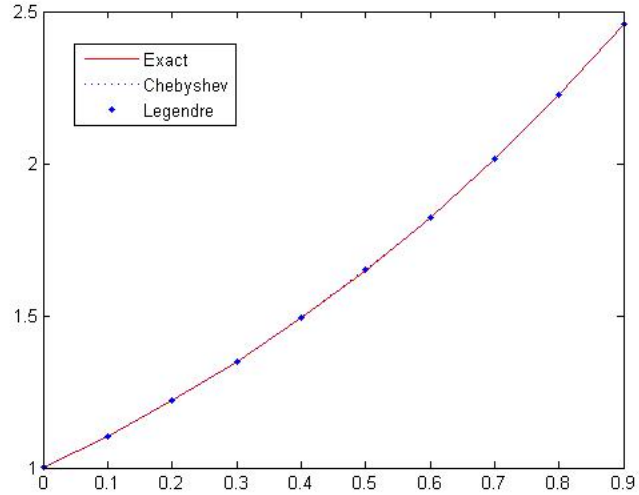

Fig. 5. Plot of exact and numerical solution for Example 4.5

Example 4.6. Solve the following Fredholm integral equation

$$\int_0^1 \frac{(t-x)^2}{1+t^2} u(t)dt = 0.179171 - 0.532108x + 0.487495x^2, \quad 0 \le x \le 1.$$

Exact solution to this problem is $u_{ex} = \sqrt{x}$.

Here both wavelets give same results and attains minimum rms given by $4.81549806e-02$ at $\alpha = 10^{-3}$. The results obtained are presented in Table 6 and Fig. 6.

Table 6. Numerical results for Example 4.6

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\lvert u_{ex}(x) - u(x)\rvert$	Error using Legendre wavelet $\lvert u_{ex}(x) - u(x)\rvert$
0	0.00000000	1.36960097e-01	1.36960097e-01
0.1	0.31622776	2.80533570e-02	2.80533570e-02
0.2	0.44721359	2.37570634e-02	2.37570634e-02
0.3	0.54772255	4.91609102e-03	4.91609102e-03
0.4	0.63245553	1.37686800e-02	1.37686800e-02
0.5	0.70710678	2.66029878e-02	2.66029878e-02
0.6	0.77459666	3.06664680e-02	3.06664680e-02
0.7	0.83666002	2.42242901e-02	2.42242901e-02
0.8	0.89442719	6.14611643e-03	6.14611643e-03
0.9	0.94868329	2.43531886e-02	2.43531886e-02

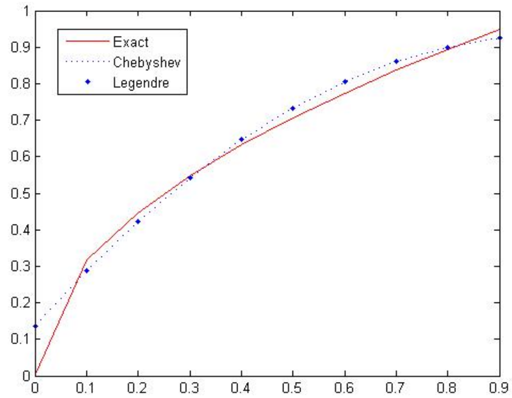

Fig. 6. Plot of exact and numerical solution for Example 4.6

In present method the error is large only at point $x = 0$ and at all other points

$$\|u - u_{ex}\|_\infty < 1.66074364e - 02.$$

From the results obtained, we can see that the method provides good results except around point $x = 0$.

Example 4.7. Solve the nonlinear Fredholm integral equation of the first kind

$$\int_0^1 xtu^3(t)dt = x/5, \quad 0 \leq x \leq 1,$$

with exact solution $u(x) = x$.

Table 7. Numerical results for Example 4.7

x	Exact solution $u_{ex}(x)$	Error using Chebyshev wavelet $\lvert u_{ex}(x) - u(x)\rvert$	Error using Legendre wavelet $\lvert u_{ex}(x) - u(x)\rvert$
0	0.00000000	4.85811927e-06	3.61861628e-05
0.1	0.39148676	3.91486686e-08	3.91408483e-05
0.2	0.49324241	4.93242315e-08	4.93143788e-05
0.3	0.56462161	5.64621505e-08	5.64508717e-05
0.4	0.62144650	6.21446376e-08	6.21322238e-05
0.5	0.66943295	6.69432816e-08	6.69299098e-05
0.6	0.71137866	7.11378518e-08	7.11236418e-05
0.7	0.74888723	7.48887089e-08	7.48737494e-05
0.8	0.78297352	7.82973371e-08	7.82816969e-05
0.9	0.81432528	8.14325121e-08	8.14162459e-05

Here, with Chebyshev wavelet the minimum value of rms=1.53753862e-06 is attained at $\alpha = 10^{(-7)}$. Legendre wavelet gives rms=6.34203116e-05 at $\alpha = 10^{(-4)}$. Table 7 and Fig. 7 shows the absolute errors obtained using both wavelets.

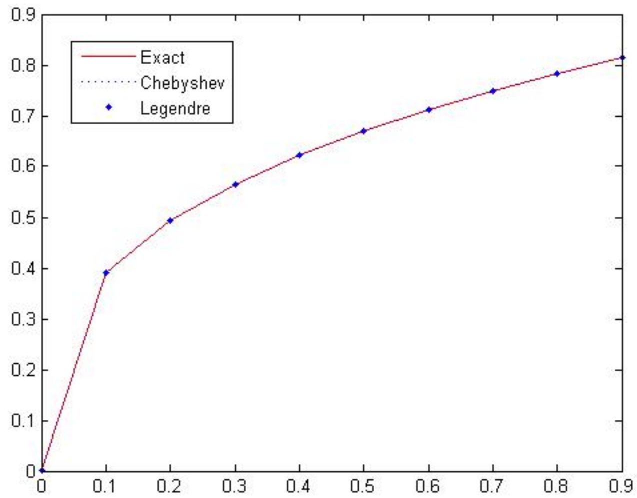

Fig. 7. Plot of exact and numerical solution for Example 4.7

5 Conclusion

In this paper, approximate solution to linear Fredholm integral equations of the first kind is derived by combining regularization and collocation method based on Legendre and Chebyshev wavelets. Both the wavelets are applied to some examples and results are presented in tables. Absolute error and rms value are calculated for both wavelets. The results obtained shows efficiency of the proposed method. After analyzing results we have found that both the wavelets give accurate results but Chebyshev wavelet gives better results than Legendre wavelet.

Competing Interests

Authors have declared that no competing interests exist.

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