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Brief Review on Asset Selection and Portfolio Construction: Diversification, Risk and Return

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Abstract

This paper covers the rationale behind Modern Portfolio Theory and discusses the assumptions of the Capital Asset Pricing Model. The investor's preference regarding financial instruments, expected return and level of risk are essential to determining the composition of the portfolio. The practice of diversification provides a solution by, the allocation of wealth into selected securities which cumulatively offer, a portfolio that produces a desirable return with minimal variance. Taking into account the investor's level of risk aversion, allows that efficient diversified portfolio to maximize his utility. The key parameter of portfolio construction, denominated by the Greek letter beta, is systematic risk which is undiversifiable. Estimating the efficient diversified portfolio's performance is dependent on that single coefficient. The risk-return trade off relationship derived from beta, provides the framework for the pricing of assets held in competitive markets. An empirical study illustrating diversification, risk preference and asset pricing provides practical assessment of the concepts developed in the paper.

Keywords

Investment, Diversification, Portfolio, Asset, Beta, Risk

1. Introduction

By exploring the different sets of asset allocation available to the investor, we can show diversification achieves the most desirable outcome (Rubinstein, 2002). The investor's objective to maximize return means the highest paying interest (bond) dividend (stock) bearing securities are first in mind for construction of his portfolio. The efficient market hypothesis makes it unlikely to significantly profit from price fluctuations. The portfolio will therefore be a long-term investment. The uncertainty that can arise from combined systematic and non-systematic risks will lead the investor to adopt a mind-set towards risk. Causing a recalibration of the investor's aim, does he intend to still maximize return regardless of risk, minimize risk for a guaranteed return or mitigate risk for an optimal return?

An investor resolute in maximizing return regardless of risk estimates is playing "Russian roulette"; the portfolio will produce extremely high return until it collapses as a result of the high level of risk. The consequence will be the loss of the portfolio's overall value, which is probably more than the cumulative gain in interests or dividends. The alternative could be "to play it safe" by investing in very low risk securities that will constitute a riskless portfolio, guaranteeing the investor a low but constant return on investment over long periods. There is a third possibility: it involves selecting various securities bearing different levels of risk, in order to diminish the overall risk[1] of the portfolio by investing different level of wealth in each and achieve the desirable expected return.

2. Markowitz's Logic

Markowitz (1952) puts forth an 'Efficient Diversification of Investments', by hypothesis of the expected return-variance of return rule. He considered the expected value of the portfolio to be the sum of all securities expected returns taking into account the wealth invested in each. He defines the covariance and variance, which are measures of dispersion as appropriate to determine risk. While Co-movements suggests correlation, the covariance expresses securities co-movements away from their mean returns; it is the systematic risk affecting all assets. The covariance of the securities forms the variance of the expected return from the portfolio and constitutes the overall risk associated with holding the investment. The investor can seek in this scenario to select a portfolio, in the terms of expected return maximization and variance minimization.

The sets of portfolios available under the conditions determined by Markowitz can be represented in a two dimensional geometry (**Figure 1**), with isomean lines for the set of expected returns on portfolios and isovariance curves for the set of their variance.

A change in expected return E means a change in the intercept but not the slope of the isomean lines depicted by the parallels. X is the point that minimizes the variance of return; it lies at the centre of the isovariance ellipses that display areas of increasing variances. It is also the point that maximizes the expected return for the lowest possible level of overall risk for the portfolio (X_1, X_2). In this regard, the expected return and variance values at point X constitute the most efficient portfolio set of this case.

Tobin (1958) suggests that all investors would find the efficient portfolio desirable but their preferences toward risk will lead them to invest their wealth differently. Consequently, the separation principle explains why in equilibrium the market portfolio will not be the most efficient portfolio set. Investors adopt vari-

[1]Williams (1938) considered risk could be "nil" supported by Bernoulli's (1713) law of large numbers, Markowitz (1952) makes the sound proposition that it is possible to only go so far as to reduce it to the single systematic risk factor.

ous behaviour towards risk depending on their life experience, professional and financial positions as well as institutional factors such as corporate directives, bonuses and taxes. In this case, the level of risk aversion will decide the weights in the efficient portfolio; making for a myriad of possibilities (**Figure 2**).

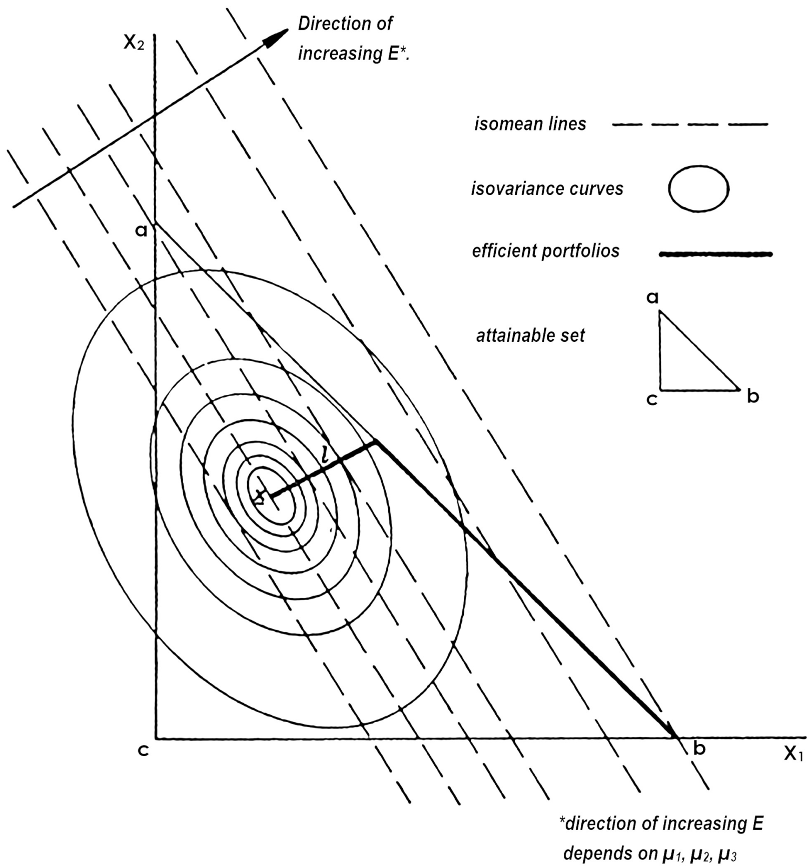

Figure 1. Mean-Variance efficient set determination (Markowitz, 1952: p. 85).

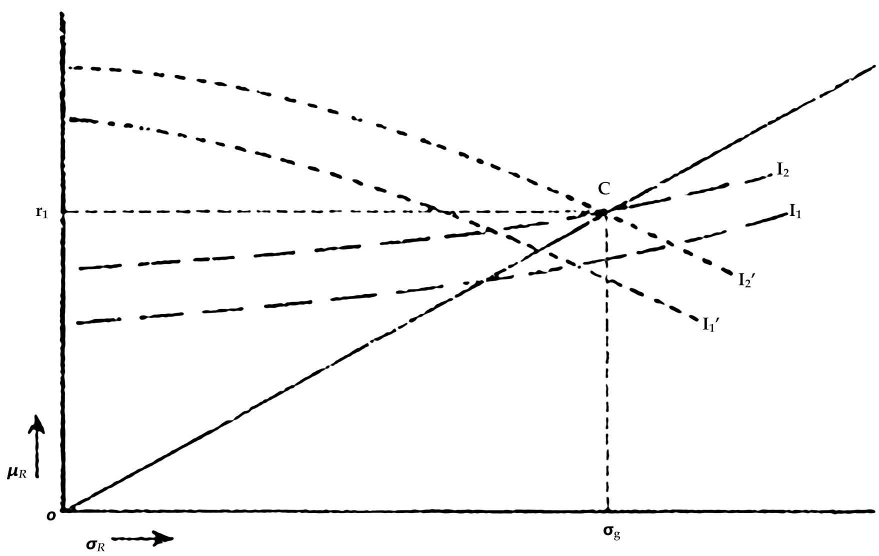

Figure 2. Utilitarian optimal portfolio selection (Tobin, 1958: p. 78).

The opportunity locus OC shows efficient sets with increasing risk σ_R and return μ_R on C as point maximising utility for all. The indifference curves display the investor utility function depending on his risk preference from underlying concerns about interest rate, tax and capital gains. The risk-lovers (I_1', I_2') will expect greater capital gains σ_g by higher variance of return, accepting lower mean return. While risk-averters (I_1, I_2) will only consider higher variance of return for substantial increase in mean return.

3. Determining Beta

Before The Greek letter (β) refers to the true value of the B parameter from Sharpe's (1963) diagonal model; its estimate is the coefficient commonly used as a measure of systematic risk. Beta is primarily the slope for the linear relationship between security return and market return. It measures the change in portfolio expected return in response to change in market volatility; this is the case of the market model. Industry and macroeconomic levels systematic risk factors impact measurements using beta appear in very extensive models[2].

Proving beta's usefulness as a measure of non-diversifiable risk can provide a clear explanation of its importance to investors. Then from the diagonal model's lending portfolio, we can assess how the investors can optimize their portfolio.

We can derive beta as a measure of risk from the portfolio approach based on Markowitz determination of risk by the measures of dispersion. The market model describes a linear relationship in which the return on the portfolio is equal to the wealth invested in risk free and risky securities. The wealth invested in the risk free security results is a constant, while the return on the risky securities is an enigma, decomposed as wealth invested in "undiversifiable" risk and "diversifiable" risk (Marshall, 1971: p. 3).

$$Var\left(\tilde{w}_t\right)=\left(\sum_{i=1}^{n}\frac{1}{n}\beta_i\right)^2 Var\left(\tilde{M}_t\right)+\sum_{i=1}^{n}\left(\frac{1}{n}\right)^2 Var\left(\tilde{\varepsilon}_{it}\right)$$

As proposed by Blume (1971), the diversifiable risk denoted by the error term in the second part of the equation will grow smaller as the investor's form a more efficiently diversified portfolio. This will reduce the model to a single (undiversifiable) risk factor denoted by beta (β), it will be then the only explanatory variable of the portfolio expected return.[3]

Now let us consider the diagonal model that makes the proposition of a portfolio composition, with wealth invested in securities on hand and wealth lent out

[2]Multiple factor models exist (Fama & French, 2018) in which multiple betas measure the reactions of portfolio return to changes in value (high-low B/M), size (Market Cap), profitability (high/low probability diversified portfolio returns) and investment patterns (high/low investment firms diversified portfolio returns). Uncertainty arising from gross national product, inflation and interest rates are includable. The surprise effect of these factors has an impact on security return captured by the beta coefficients.

[3]"In effect, the market presents him with two prices: the price of time, or the pure interest rate…and the price of risk" (Sharpe, 1964: p. 425). That is to say, the value of any asset follows the logic set in the Gordon-Shapiro (1956) equation, as interest rate measure the impact of time. It also follows Markowitz (1952) mean-variance rule, as variance captures the influence of risk.

on the other. Of course, the selection of the securities will follow the mean-variance rule, constituting an efficient diversified portfolio. The wealth invested in the lending scheme guarantees an interest rate and principal repayment at the end; consider it a portfolio in itself. There is no risk involved in the lending portfolio that means there is no variance in return to be expected. The investor is able to optimize expected portfolio return by selecting a combination of both portfolios, not just holding the efficient diversified portfolio. This is because the hypothesis of a lending portfolio will affect the sets of available efficient portfolios making many of them inefficient.

In this **Figure 3**, where the curve *FG* represents the sets of efficient portfolios by the two dimensions *E*, σ.

H is a tangent created by a lending portfolio (r_l) giving *AB* as the line containing the only sets of efficient portfolios, leaving the rest of the sets on *FB* inefficient. In this case, for a low level of variance a risk averse investor can achieve a relatively high level of return by combining the efficient portfolio and the lending portfolio at point *B*.

D is a tangent created by an efficient portfolio *C* with a corresponding lending portfolio (r_b), leaving the rest of the sets on *FC* inefficient. In this case, an investor seeking to maximize return can choose the combination efficient portfolio and lending portfolio at point *C* as a start and continue to do so by selecting other sets on *CD*.

Fama and MacBeth (1973) provides empirical testing with as proposition the hypothesis of a positive relationship between realized portfolio returns and

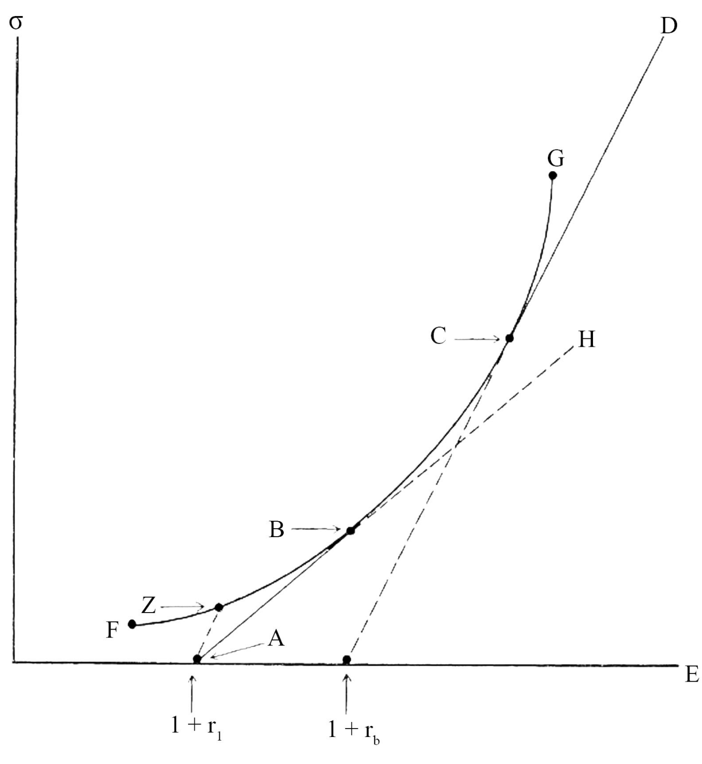

Figure 3. Diagonal model dual portfolio optimization (Sharpe, 1963: p. 286).

betas. It was able to establish a systematic relation between both variables and confirm the positive risk return trade off. The results have been conditional on the performance of the market as it shows. The investor was able to benefit from risk premium, when realized market return was higher than risk free return; with positive excess returns. This was the case of high betas portfolios, meaning high covariance of return between the investor's portfolio and the market. However, when market return was lower than the risk free return; low betas portfolios outperformed high betas when comparing their portfolio returns. The investor holding a risky portfolio, in this case experienced negative excess returns. Based on our approaches, we can conclude that the beta estimate in the context of market conditions can guide the investor's decision to allocation of wealth and indicate portfolio performance.

4. The Risk Return Trade off

In order to construct the portfolio that will yield the desirable return, it is necessary to value each asset that can form it. We can define the price of any asset as the expected return of that asset as result of the combination[4] of time and risk involved in holding that asset. The latter is of great importance in competitive markets. The following models enable a measure of the price of risk for any security by expected return estimation.

The Capital Asset Pricing Model builds on a model of investor behaviour seeking to maximize utility while minimizing variance of return. The efficient frontier resulting from behavioural assumptions represents the spectrum of the investment opportunities. He can choose to maximize his utility by choosing any point along the investment opportunity curve depending on risk aversion. The point of tangency with the investor's indifference curve is the portfolio set that maximizes his utility. The wealth allocation necessarily follows Tobin (1958) and Hicks (1962). It is possible to form the desirable portfolio as a combination, with lending and borrowing at a risk free rate.

Under the above conditions, Sharpe (1964) derives an equilibrium of assets prices as investors utility choices lead fluctuations in the price of securities until the investment opportunity curve becomes linear, the capital market line. Every feasible combination between an asset in the geometrical plane and a portfolio set on the line can be modelled using regression analysis; observed data would show a relation between the asset's return variance and covariance with the combination return, meaning it is possible to measure the systematic risk of every single asset, and then predict its expected return and therefore its price. Lintner (1965) makes that proposition explicit, by using the S&P Index as proxy for the market as a whole and we can draw the following conclusions.

"In accordance with these relationships, investors will regard the expected rate

[4] Allowing the presence of residual variances is adding to systematic risk the possibility of slightly heterogeneous beliefs of investors about the future Lintner (1965). The investor's portfolio is to benefit, from the relative positive correlation of the assets with the market; taking full advantage of diversification. In addition, the equilibrium of stock prices as it is requires this assumption.

of return on any i'th stock as

$$\overline{r} = a_i + b_i \overline{I}$$

and its expected excess return as (Lintner, 1965: p. 602)

$$\overline{X}_i = \overline{r}_i - r^* = a_i + b_i \overline{I} - r^* \text{"}$$

In equilibrium, the value of any stock is likely to vary depending on its initial payout denoted by constant a_i and the covariance of the stock b_i with the Index variance of return I, implying that residual variance is not an explanatory variable of the stock expected return. With a risk free return r^* possible from the Index, there can be excess in return on the stock.

In equilibrium, the price of any stock may vary depending on the effects of the change in the correlation coefficient, ceteris paribus. It is possible to observe an income effect when there is an unexpected increase in the asset realised return because of increased variance. In this case, the investor's asset yields a higher return considering its beta, leading investors to purchase similar assets on the market resulting in overall increase of prices for the asset. A risk effect is observable when an unexpected increase in the asset variance means a lower than expected realised return on the asset. In this case, the investor's asset has a higher risk compared to its expected return, leading to disinvestment from all investors on this particular asset resulting in decrease in prices.

The Arbitrage Pricing Model in terms of assumptions does not consider the mean variance rule necessary, but remains attach to the risk-return trade off with beta and supports the idea of diversification with the possibility of arbitrage. Ross (1976) makes the proposition of a diversified arbitrage portfolio characterised by no initial wealth investment, no risk and positive expected return. This is possible in inefficient markets, with the investor taking a short and long position on the same asset or a similar one. The very risk averse investor can maximize his utility by holding in this arbitrage portfolio only riskless asset earning him (E_i) and the less risk averse investor can benefit from the beta vector (β_{ik}) as return in holding the arbitrage portfolio with risky asset can increase with factors (δ_k) (Ross, 1976: p. 347)

$$\begin{aligned} \tilde{x}_i &= E_i + \beta_{i1}\tilde{\delta}_1 + \cdots + \beta_{ik}\tilde{\delta}_k + \tilde{\epsilon}_i \\ &\equiv E_i + \beta_i \tilde{\delta} + \tilde{\epsilon}_i \end{aligned}$$

where

$$E\left\{\tilde{\delta}_i\right\} = E\left\{\tilde{\epsilon}_i\right\} = 0$$

The return on the portfolio is therefore subject to the explanatory variables of a factor model similar to CAPM with the residual variance (ϵ_i) playing no role. However, different as there is no single systematic risk factor but rather a vast probability of risks that can affect return. These risks can range from macroeconomics factors such as inflation, growth to microeconomic factors such as return on equity and commodity price.

Doubt over the practical use of CAPM already surface in Lintner (1965) with scrutiny on Sharpe's limit case of zero residual variance. It is worth mentioning that APT relies heavily on it, while casting aside the fundamental assumption of market efficiency. Conveniently so, since rapid adjustments ensure mispricing on stock markets is only temporary, making the Arbitrage is also a limit case. The hypothesis that any asset price can only result from expectations in future returns is essential, total inability for CAPM to estimate expected return because of risk factors completely independent from the market cannot possibly be. Merton (1973) intertemporal CAPM addresses several of the criticisms and subsequent empirical study (Merton, 1985) was able to show that the market rationality hypothesis implied by CAPM remains valid[5] when investigating historical data.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Bernoulli, J. (1713). *Ars Conjectandi (Thurnisorium, Basil).*

Blume, M. E. (1971). On the Assessment of Risk. *The Journal of Finance, 26,* 1-10. https://doi.org/10.1111/j.1540-6261.1971.tb00584.x

Fama, E. F., & French, K. R. (2018). Choosing Factors. *Journal of Financial Economics, 128,* 234-252. https://doi.org/10.1016/j.jfineco.2018.02.012

Fama, E. F., & MacBeth, J. D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy, 81,* 607-636. https://doi.org/10.1086/260061

Gordon, M. J., & Shapiro, E. (1956). Capital Equipment Analysis: The Required Rate of Profit. *Management Science, 3,* 102-110. https://doi.org/10.1287/mnsc.3.1.102

Hicks, J. R. (1962). Economic Theory and the Evaluation of Consumers' Wants. *The Journal of Business, 35,* 256-263. https://doi.org/10.1086/294509

Kahneman, D., & Tversky, A. (1979). On the Interpretation of Intuitive Probability: A Reply to Jonathan Cohen. *Cognition, 7,* 409-411. https://doi.org/10.1016/0010-0277(79)90024-6

Kahneman, D., & Tversky, A. (1982). The Psychology of Preferences. *Scientific American, 246,* 160-173. https://doi.org/10.1038/scientificamerican0182-160

Lintner, J. (1965). Security Prices, Risk, and Maximal Gains from Diversification. *The Journal of Finance, 20,* 587-615. https://doi.org/10.1111/j.1540-6261.1965.tb02930.x

Markowitz, H. (1952). Estimation Risk in the Portfolio Selection Model. *The Journal of Finance, 7,* 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

Marshall, B. (1971). On the Assessment of Risk. *Journal of Finance, 26,* 1-10. https://doi.org/10.1111/j.1540-6261.1971.tb00584.x

Merton, R. C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica: Journal of the Econometric Society, 41,* 867-887. https://doi.org/10.2307/1913811

[5]On the back of the work of Tirole (1982) on speculative bubbles, Kahneman and Tversky (1979, 1982) highlighting human error in decision making and Shiller, Fischer, & Friedman (1984) going as far as considering the irrationality in investor behaviour which all explicitly more or less, question the validity of assumptions underlying Modern Portfolio Theory.

Merton, R. C. (1985). *On the Current State of the Stock Market Rationality Hypothesis* (No. 1717-85). Massachusetts Institute of Technology (MIT), Sloan School of Management.

Ross, S. A. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory, 13,* 341-360. https://doi.org/10.1016/0022-0531(76)90046-6

Rubinstein, M. (2002). Markowitz's "Portfolio Selection": A Fifty-Year Retrospective. *The Journal of Finance, 57,* 1041-1045. https://doi.org/10.1111/1540-6261.00453

Sharpe, W. F. (1963). A Simplified Model for Portfolio Analysis. *Management Science, 9,* 277-293. https://doi.org/10.1287/mnsc.9.2.277

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance, 19,* 425-442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Shiller, R. J., Fischer, S., & Friedman, B. M. (1984). Stock Prices and Social Dynamics. *Brookings Papers on Economic Activity, 1984,* 457-510. https://doi.org/10.2307/2534436

Tirole, J. (1982). On the Possibility of Speculation under Rational Expectations. *Econometrica: Journal of the Econometric Society, 50,* 1163-1181. https://doi.org/10.2307/1911868

Tobin, J. (1958). Liquidity Preference as Behavior towards Risk. *The Review of Economic Studies, 25,* 65-86. https://doi.org/10.2307/2296205

Williams, J. B. (1938). *The Theory of Investment Value* (Vol. 36). Harvard University Press.

Appendix

The purpose of this empirical study is to illustrate, the complementary positions on portfolio construction proposed in this paper.

Let's consider the mean-variance optimization problem without going into numerous constraints that may arise from utility functions. Instead, let's look into the two extreme cases, risk minimization and expected return maximization.

We will use actual securities that are constituent of the Standard & Poor's 500 Index: Apple, Merck & Co, Procter & Gamble and Nike.

The data set consists of 1 year daily price (05/10/2022-05/10/2023), with return calculated on the basis of adjusted close values, variance/covariance matrix is built (**Table A1**). Then daily average price change, population standard deviation and variance are calculated and these variables are used to compute annual return and volatility (**Table A2**) necessary for optimization problems. Finally, a portfolio built on the assumption of equal weight distribution is built (**Table A3**) for comparison with optimization solutions.

Table A1. Securities covariance matrix.

Covariance Matrix	APPLE	MERCK & CO	P&G	NIKE
APPLE	0.000294823	2.89771E−05	6.6945E−05	0.000162332
MERK	2.89771E−05	0.000155087	6.6945E−05	0.000162332
P&G	6.6945E−05	4.75503E−05	9.09139E−05	1.07188E−05
NIKE	0.000162332	1.07188E−05	7.35739E−05	0.00037281

Table A2. Securities measures of dispersion.

Dispersion Measures	APPLE	MERCK	P&G	NIKE
Daily Price Change	0.09%	0.08%	0.05%	0.05%
Daily Std. Deviation	0.01713646	0.01242877	0.009527595	0.01934633
Daily Variance	0.01934633	0.00015447	9.07751E-05	0.00037428
Annual Return	24.93%	22.78%	14.73%	14.61%
Annual Volatility	0.27203287	0.19730068	0.151245888	0.30711351

Table A3. Portfolio under assumption of equal weight distribution.

Solver generated						
Portfolio Weight	0.25	0.25	0.25	0.25	Weight Constraint	1
Individual securities	APPLE	MERCK & CO	P&G	NIKE	Expected return	4.82%
Expected Return	6.23%	5.69%	3.68%	3.65%	Portfolio Variance	0.000105864
Return Volatility	0.0680082	0.0493252	0.0378115	0.0767784	Portfolio Volatility	0.010289034

A.1. The Diversification Proposition Markowitz (1952)

In order to achieve diversified asset selection the first proposition is necessary, the portfolio is built solely for the purpose of maximizing expected return for the lowest possible level of risk.

This would be the case of risk minimization (Table A4), where the desirable portfolio is determined by, minimizing Return Variance subject to a non-negative minimum expected return as well as constraint on wealth allocation with each security weight in the portfolio being non-negative. The original value of variance is based on the equal distribution of weight in the portfolio (Table A3). The final value of variance resulting from solving the risk minimization is much lower, the percentage change was calculated to equal −26.98%.

The results in Table A5 show wealth allocation concentration with Merck & Co (30%) and P&G (50%) in portfolio weight, this may be explained by the relative higher expected return to return volatility of these securities compared to APPLE and NIKE. Indeed, the latter two offer respectively higher annual volatility of returns (0.27; 0.30) against Merck & Co and P&G (0.19; 0.15) as displayed in Table A2.

Table A4. Solving risk minimization.

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Solver Options

Objective Cell (Min)

Cell	Name	Original Value	Final Value	%Change
$Q$20	Return Variance	0.000105864	7.73064E−05	−26.98%

Variable Cells

Cell	Name	Original Value	Final Value	Lagrange Multiplier
$J$18:$M$18				
$Q$20	Portfolio Weight	1	1	0.000147834

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$18	Weight	1.00	$Q$18 = 1	Binding	0
$J$18	Portfolio Weight APPLE	0.10	$J$18 ≥ 0.1	Binding	0.00
$K$18	Portfolio Weight MERCK	0.30	$K$18 ≥ 0.1	Not Binding	0.20
$L$18	Portfolio Weight P&G	0.50	$L$18 ≥ 0.1	Not Binding	0.40
$M$18	Portfolio Weight NIKE	0.10	$M$18 ≥ 0.1	Binding	0.00

Table A5. Solved risk minimization.

Solver generated						
Portfolio Weight	0.1	0.296706824	0.503293166	0.1	Weight Constraint	0.99999999
Individual securities	APPLE	MERCK	P&G	NIKE	Portfolio Expected return	6.13%
Expected Return	2.49%	6.76%	7.41%	1.46%	Portfolio Variance	7.73064E−05
Return Volatility	0.02720329	0.05854046	0.07612102	0.03071135	Portfolio Volatility	0.008792405

A.2. The Investor's Risk Preference Proposition James Tobin (1958)

In order to achieve wealth allocation that suits investor's risk tolerance the second proposition is important, the efficient portfolio established through the first proposition in Table A5; can be altered in terms of expected return by changing the level of risk parameter.

This would be the case of expected return maximization (Table A6), where the desirable portfolio is determined by, maximizing Expected Return, subject to constraint on wealth allocation with each security weight in the portfolio being non-negative. The original value of expected return is based on the equal distribution of weight in the portfolio consisting of the four securities. The final value of expected return resulting from solving the expected return maximization is much higher, the percentage change was calculated to equal +164.53%.

The results in Table A7 show wealth allocation concentration in APPLE (70%) with remaining portfolio weight distributed equally to the three other securities. This may be explained by APPLE offering the highest possible expected return (24.93%) from all securities in the portfolio (see Table A2). When compared to the efficient portfolio from the first proposition (Table A5), expected return has literally doubled from 6.13% to 12.74% or a 6.61% increase. The risk return trade off can be observed from the portfolio variance that has gone up 2.45 times compared to its estimate in the efficient portfolio (7.73E−05/0.000189).

Table A6. Solving expected return maximization.

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Solver Options

Objective Cell (Max)

Cell	Name	Original Value	Final Value	%Change
$Q$19	Expected return	0.048150701	0.127373427	164.53%

Variable Cells

Cell	Name	Original Value	Final Value	Lagrange Multiplier
$J$18: $M$18				
$Q$18	Portfolio Weight	1	1	0.349035736

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$18	Weight	1.00	$Q$18 = 1	Binding	0
$J$18	Portfolio Weight APPLE	0.70	$J$18 ≥ 0.1	Not Binding	0.60
$K$18	Portfolio Weight MERCK	0.10	$K$18 ≥ 0.1	Binding	0.00
$L$18	Portfolio Weight P&G	0.10	$L$18 ≥ 0.1	Binding	0.00
$M$18	Portfolio Weight NIKE	0.10	$M$18 ≥ 0.1	Binding	0.00

Table A7. Solved expected return maximization.

Solver Generated						
Portfolio Weight	0.7	0.1	0.1	0.1	Weight Constraint	1
Individual Securities	APPLE	MERCK	P&G	NIKE	Portfolio Expected return	12.74%
Expected Returns	17.45%	2.28%	1.47%	1.46%	Portfolio Variance	0.000189444
Return Volatility	0.190423	0.0197301	0.0151246	0.0307114	Portfolio Volatility	0.013763853

Implication for Capital Asset Pricing Model (CAPM)

Diversification greatly simplifies the mathematical model accordingly with Lintner (1965); by making the error term extremely small and one could say in some instances insignificant. This implies the pricing of securities is subject only to a function of market change denoted by the term beta, Sharpe (1964). Computing the expected return from the portfolio would be reduced to the simple expression of the wealth allocated in the securities whose returns are determined by market change.

Application of the Capital Asset Pricing Model, is done by using 5 year daily returns (09/10/2018-06/10/2023) of APPLE, Merck & Co, Procter & Gamble, NIKE and the addition of the S&P 500. The returns are calculated from daily price adjusted close values, the covariance matrix is built with each security returns and the market returns (**Table A8**).

The covariance matrix (**Table A8**) enables us to estimate the beta of each security, using the covariance of each security with the market and market variance. In **Table A9**, the data shows that APPLE and NIKE are more volatile and in positive co-movement with the S&P 500 Index (respective beta; 1.23 and 1.06). While Merck & Co and P&G have been far less volatile and only in partial co-movement with the market with beta of 0.52 and 0.57.

Expected return of each security has been calculated using 5 year Treasury Yield and S&P 500 Yield as of October 6th 2023. Higher beta value is associated with higher expected returns when compared to the market, APPLE and NIKE (9.19%; 8.58%) are outperforming the S&P 500 Index (8.35%). While Merck & Co and P&G are providing below market expected returns (6.68%; 6.83%), however they remain higher than the risk free rate (treasury yield is only 4.76%).

Portfolio beta and return will be calculated under the assumption of equal weight distribution (**Table A3**) as well as under the two propositions (**Table A5** and **Table A7**).

To do so, the only necessary assumption is to adjust the weight in the portfolio to match solutions found through optimization.

Be advised, that this method might not lead to most accurate results; because these solutions resulted from only 1 year data while asset pricing data consisted of 5 year observations. However, it could be indicative since the solutions are based on 2022-23 observations, while the asset pricing model was built using historical data 2018-2023.

Table A8. Securities covariance matrix (5 year daily Returns 2018-2023).

Covariance Matrix					
Securities	AAPL	MKR	PG	NKE	^GSPC
AAPL	0.000440807	0.00010677	0.000124882	0.000238556	0.000233274
MKR	0.00010677	0.000215606	9.21964E−05	9.07593E−05	0.000101078
PG	0.000124882	9.21964E−05	0.000186896	0.000112561	0.00010911
NKE	0.000238556	9.07593E−05	0.000112561	0.000439656	0.000201065
^GSPC	0.000233274	0.000101078	0.00010911	0.000201065	0.000188956

Table A9. Securities beta and expected return.

Securities	APPLE	MRK	P&G	NIKE	S&P500	As of October 6th 2023	
Beta	1.234544667	0.534930257	0.577439372	1.064088015	1	Treasury Yield 5 Year	4.76%
Expected Return	9.19%	6.68%	6.83%	8.58%	8.35%	S&P 500 Yield 5 Year	8.35%

Under the assumption of equal weight distribution in the securities (Table A10), APPLE and NIKE would provide the highest returns (2.30%; 2.15%), cumulatively the portfolio will earn the investor 7.82%; which is performance below market (8.35%) but above risk free rate (4.76%). Portfolio beta (0.85) indicates it would be performing below market volatility, while the Sharpe ratio (2.43) confirms the investment will generate significant excess return relative to risk taking.

Under the assumption of weight distribution obtained through risk minimization (Table A11), Procter & Gamble would provide the highest return (3.44%), cumulatively the portfolio will earn the risk averse investor 7.20% slightly lower than under assumption of equal weight distribution (7.82%; Table A10). This would performance below market (8.35%) but above risk free rate (4.76%). Portfolio beta (0.67) indicates it would be performing well below market volatility, while the Sharpe ratio (1.94) confirms the investment will generate significant excess return relative to risk taking.

Under the assumption of weight distribution obtained through return maximization (Table A12), APPLE would provide by far the highest return (6.43%), cumulatively the portfolio will earn the risk lover investor 8.64%, which is 1.44% more than the risk averse investor's return (7.82%; Table A11). This would be performance just above market (8.35%) and well above risk free rate (4.76%). Portfolio beta (1.08) indicates it would be performing in close co-movement with market volatility, while the Sharpe ratio (3.09) confirms the investment will generate the largest excess return relative to risk taking of all three scenarios (Tables A10-12).

Implication for Arbitrage Pricing Theory Model (APTM)

On top of creating a model based on diversification, the APTM gives the possibility for the investor to effectively hedge its exposure to specific market conditions; by measuring risk factors associated with securities. Instead of relying on the single systematic risk measure by market change, Sharpe's beta is deconstructed by using risk factors that are based on economic indicators. The estimation of these betas creates a greater challenge for the mathematical modelling of expected return.

Application of APTM, is done with a data set consisting of the same securities but with 9.5 year monthly returns (01/12/2013-01/06/2023), as well as the addition of the S&P 500, Consumer Price Index (CPI), Money Supply (M2SL), Industrial Production (INDPRO) and U.S. 1 Month Treasury Bill (DGS1MO).

The factors data are ran using linear regression against each individual security returns, in order to find the corresponding beta (Table A13). The statistical significance of each beta is interpreted from the regression summary output (Tables A14-A17).

Table A10. CAPM portfolio under assumption of equal weight distribution.

Securities		APPLE	MERCK & Co	P&G	NIKE
Portfolio Weight Assumption	1	0.25	0.25	0.25	0.25
Securities Expected Return		2.30%	1.67%	1.71%	2.15%
Portfolio Expected Return	7.82%				
Securities Beta		0.308636167	0.133732564	0.144359843	0.266022004
Portfolio beta	0.852750578				
Sharpe Ratio	2.437305				

Table A11. CAPM portfolio under assumption of risk minimization.

Securities		APPLE	MERCK & Co	P&G	NIKE
Portfolio Weight Assumption	0.99999999	0.1	0.296706824	0.503293166	0.1
Securities Expected Return		0.92%	1.98%	3.44%	0.86%
Portfolio Expected Return	7.20%				
Securities Beta		0.123454467	0.158717457	0.29062129	0.106408802
Portfolio Beta	0.679202016				
Sharpe Ratio	1.9412739				

Table A12. CAPM portfolio under assumption of return maximization.

Securities		APPLE	MERCK & Co	P&G	NIKE
Portfolio Weight Assumption	1	0.7	0.1	0.1	0.1
Securities Expected Return		6.43%	0.67%	0.68%	0.86%
Portfolio Expected Return	8.64%				
Securities Beta		0.864181267	0.053493026	0.057743937	0.106408802
Portfolio Beta	1.081827031				
Sharpe Ratio	3.0920441				

Table A13. APTM risk factors beta.

Regression Coefficient	APPL	MRK	PG	NKE
SP500_PCH	0.010201105	0.000947565	0.00325375	0.007692842
M2SL_PCH	0.014367836	−0.0064266	0.000832175	0.006782602
INDPRO_PCH	0.00199692	−0.00399414	−0.00023024	0.002696276
CPALTT01USM657N	0.01438796	0.016590476	−0.01729686	−0.03000956

Table A14. APPLE regression.

SUMMARY OUTPUT	APPLE					
Regression Statistics			Coefficients	Std Error	t Stat	*P*-value
Multiple R	0.465054302	Intercept	0.004274768	0.009993176	0.427768701	0.669655825
R Square	0.216275504	SP500_PCH	0.010201105	0.002022161	5.044654211	1.80681E-06
Adjusted R Square	0.187776431	M2SL_PCH	0.014367836	0.008967947	1.602132157	0.111994255
Standard Error	0.072484551	INDPRO_PCH	0.00199692	0.004877138	0.40944499	0.683009322
Observations	115	CPALTT01USM657N	0.01438796	0.019735582	0.729036548	0.467528707

Table A15. Merck & Co regression.

SUMMARY OUTPUT	MERCK & CO					
Regression Statistics			Coefficients	Std Error	t Stat	*P*-value
Multiple R	0.153039486	Intercept	0.010916395	0.00767905	1.42158152	0.157976794
R Square	0.023421084	SP500_PCH	0.000947565	0.001553888	0.609802493	0.543250281
Adjusted R Square	−0.01209088	M2SL_PCH	−0.0064266	0.006891233	−0.93257678	0.353080965
Standard Error	0.055699255	INDPRO_PCH	−0.00399414	0.003747736	−1.06574714	0.288871626
Observations	115	CPALTT01USM657N	0.016590476	0.0151654	1.093968952	0.276358392

Table A16. Procter & Gamble regression.

SUMMARY OUTPUT	Procter & Gamble					
Regression Statistics			Coefficients	Std Error	t Stat	*P*-value
Multiple R	0.280779584	Intercept	0.009455868	0.006177248	1.53075726	0.128700247
R Square	0.078837175	SP500_PCH	0.00325375	0.001249992	2.603016239	0.010514779
Adjusted R Square	0.045340345	M2SL_PCH	0.000832175	0.005543506	0.15011711	0.880947134
Standard Error	0.044806082	INDPRO_PCH	−0.00023024	0.003014786	−0.07637123	0.939262487
Observations	115	CPALTT01USM657N	−0.01729686	0.012199484	−1.41783511	0.159065503

Table A17. NIKE regression.

SUMMARY OUTPUT	NIKE					
Regression Statistics			Coefficients	Standard Error	t Stat	*P*-value
Multiple R	0.406023061	Intercept	0.009199963	0.009339787	0.985029222	0.326771396
R Square	0.164854726	SP500_PCH	0.007692842	0.001889945	4.070404551	8.8665E-05
Adjusted R Square	0.134485807	M2SL_PCH	0.006782602	0.008381591	0.809226162	0.420131997
Standard Error	0.067745256	INDPRO_PCH	0.002696276	0.004558254	0.591515144	0.555388985
Observations	115	CPALTT01USM657N	−0.03000956	0.0184452	−1.6269578	0.106607359

The factors' risk premiums are calculated for each factor considering risk free rate than the average of each risk premium series are computed. Effectively, the explanatory power of the factors (betas) and the risk premiums average are used to estimate the expected return of each security (Table A18).

The regression coefficients represent to the beta of the risk factor, reflecting the change in the security that may be explained by each economic indicator. While values were computed for each factor (Table A13), statistical significance was only valid for the market beta. The S&P500 has been found to be the only factor that has a beta with explanatory power for APPLE, Procter & Gamble and NIKE. No beta was found to be statistically significant to explain Merck & Co returns.

Looking at the statistical significance for APPLE (Table A14), only the S&P500 generated a coefficient that has explanatory power, with positive t-Stat (5.04) greater than 1.96 threshold and *P*-value (1.807E−01) indicating significance at the 5% level. Regression on the single S&P risk factor gives an R Square of 0.195, which would imply that most of the explanatory power in the multiple factor regression (0.216) is due to the market risk factor alone.

Looking at the statistical significance for Merck & Co (Table A15), no risk factor was found to have a coefficient with explanatory power when considering t-Stat and *P*-value.

Looking at the statistical significance for Procter & Gamble (Table A16), only the S&P500 generated a coefficient that has explanatory power, with positive t-Stat (2.60) greater than 1.96 threshold and *P*-value (0.01) indicating significance at the 5% level. Regression on the single S&P risk factor gives an R Square of 0.0578, which would imply that most of the explanatory power in the multiple factor regression (0.0788) is due to the market risk factor alone.

Looking at the statistical significance for NIKE (Table A17), only the S&P500 generated a coefficient that has explanatory power, with positive t-Stat (4.07) greater than 1.96 threshold and *P*-value (8.866E-05) indicating significance at the 5% level. Regression on the single S&P risk factor gives an R Square of 0.136, which would imply that most of the explanatory power in the multiple factor regression (0.164) is due to the market risk factor alone.

Table A18. APTM risk premiums and expected returns.

1M Treasury Bill	0.05423			
Factor Risk Premiums	S&P-Rf	CPI-Rf	M2-Rf	IndProd-Rf
	0.783212087	0.180902788	0.511069565	−0.02132696
Significant Coefficient	APPL	MRK	PG	NKE
SP500_PCH	0.010201105	0.000947565	0.00325375	0.007692842
	APPL	MRK	PG	NKE
Expected Returns	6.22%	5.50%	5.68%	6.03%

The factors' risk premiums are calculated for each factor considering risk free rate than the average of each risk premium series are computed. Effectively, the explanatory power of the factors (betas) and the risk premiums average are used to estimate the expected return of each security (**Table A18**).

When calculating expected returns from APTM (**Table A18**), the results of the regressions mean all risk factors except for the S&P 500 have to be omitted, in order to obtain a valid result. Effectively, it also means that it is not possible to calculate Merck & Co expected return on the basis of statistical significance.

Results suggest that what can be expected are returns in excess of risk free rate for the securities (APPLE (6.22%), Procter & Gamble (5.68%) and NIKE (6.03%)) according to the market risk factor (S&P 500 Index).

In a nutshell, this small exercise gives evidence supporting the importance of market risk, and may explain why Ross (1976) did not specify specific risk factors, because as observed; it is fairly difficult to find the indicators that would be statistically significant to support the multi factor model.

Data Sources

https://finance.yahoo.com/quote/AAPL/history?p=AAPL

https://finance.yahoo.com/quote/PG/history?p=PG

https://finance.yahoo.com/quote/MRK/history?p=MRK

https://finance.yahoo.com/quote/NKE/history?p=NKE

https://fred.stlouisfed.org/series/DGS1MO

https://fred.stlouisfed.org/series/SP500#0

https://fred.stlouisfed.org/series/M2SL#0

https://fred.stlouisfed.org/series/INDPRO/

https://fred.stlouisfed.org/series/CPALTT01USM657N#0