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# A New Method for Calculating Residual Astigmatism Produced by Toric Intraocular Lens Rotation 

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Authors' contributions
This work was carried out in collaboration between the two authors. Author YY designed the study, wrote the protocol, and wrote the first draft of the manuscript. Author TK managed the literature searches, analyzed the results, and supervised the study. Both authors read and approved the final manuscript.


#### Abstract

Aims: To report a new method for calculating residual astigmatism when exact and inexact cylinder power toric intraocular lenses (IOLs) are rotated. Methodology: Using Excel spreadsheets and $\sin ^{2} \theta$ values, we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal astigmatic powers and calculated values for various corneal astigmatisms and toric IOLs. From these values, we created charts of residual astigmatic powers and axes demonstrating the results of IOLs ranging from $50 \%$ to $150 \%$ and rotated $0^{\circ}$ to $30^{\circ}$. Results: Comparing residual astigmatic powers, the rank orders for each IOL were as follows: $100 \%<90 \%<80 \%$ at $5^{\circ}$ of rotation, $90 \%<100 \%<80 \%$ at $10^{\circ}$ of rotation, and $90 \%<80 \%<75 \%<100 \%$ at $15^{\circ}$ of rotation. Thus, lower cylinder power IOLs perform better than higher power IOLs when rotated over $5^{\circ}$. Furthermore, the residual astigmatic powers of $125 \%$ IOLs were always higher than those of $100 \%$ IOLs; however, the residual astigmatic powers of $75 \%$ IOLs became lower than those of $100 \%$ IOLs when rotation exceeded $15^{\circ}$. Conclusion: Our method shows that lower cylinder power IOLs are advantageous in environments where IOL rotation is likely and when inexact IOLs are utilized.


[^0]Keywords: Astigmatism; toric; rotation; calculation.

## 1. INTRODUCTION

Toric intraocular lenses (IOLs) are relatively effective in correcting corneal astigmatisms [1,2]. However, errors in measuring both the corneal astigmatic axis and postoperative IOL rotation may occur [3,4]. Moreover, inexact IOLs are sometimes utilized because the cylinder powers of toric IOLs are limited, while corneal astigmatic powers are infinite. Therefore, we need precise calculation methods for residual astigmatism when higher, equal, or lower cylinder power IOLs are rotated compared to the corneal astigmatic power.

In general, the astigmatic distribution curve of a cylinder lens of $f$ diopters (Cylinder (C)+f Diopter (D) $\times 0^{\circ}$ ) is represented by $f \sin ^{2} \theta$ [5]. In the present study, we used $\sin ^{2} \theta$ and Excel spreadsheets to calculate the residual astigmatism produced by toric IOL rotation. Furthermore, we expressed the cylinder powers of toric IOLs in percentage terms compared to corneal astigmatic powers. Therefore, our method can be applied to any corneal astigmatism and to any cylinder power IOL.

Several calculation methods have been previously reported. Two main types of methods have been discussed, namely vector [6,7] and matrix methods [ 8,9 ]. While these methods are detailed and precise, the calculations must be performed one-by-one. In contrast, our method allows us to perform many precise calculations simultaneously in Excel spreadsheets using percentage terms and $\sin ^{2} \theta$. Furthermore, our method also allows for visually comprehensible charts to be obtained. Consequently, we are able to decide whether a higher, equal, or lower cylinder power IOL would be most appropriate.

We would typically use the website "AcryS of Toric Calculator" when preoperatively deciding which IOL would be most optimal [10]; however, this does not explain why a particular IOL is selected. In contrast, our method allows us to better understand the concept as it sheds light on the theoretical background behind the calculations through its tables and figures.

## 2. METHODOLOGY

Firstly, we defined the corneal astigmatism as $\mathrm{C}+2.0 \mathrm{D} \times 0^{\circ}$ ( $\mathrm{f}=2.0$ ), which is represented by 2 $\sin ^{2} \theta$ (Fig. 1). Furthermore, we expressed the cylinder powers of toric IOLsin percentage terms compared to the corneal astigmatic powers. Namely, the cylinder power of a $100 \%$ toric IOL is 2.0 D , and that of a $75 \%$ toric IOL is 1.5 D if the corneal astigmatic power is 2.0 D (2.0 D $\times 1.0=2.0 \mathrm{D}, 2.0 \mathrm{D} \times 0.75=1.5 \mathrm{D}$ ).

Under these conditions, we created (Table 1) to explain our calculations. In the table, $\sin ^{2} \theta$ is calculated using the actual formula (Microsoft Excel, Redmond, WA, USA) as follows: $\sin (\theta \times \mathrm{PI}() / 180) \times \sin (\theta \times \mathrm{PI}() / 180)$

Through this formula, we can calculate values using degrees rather than radians; moreover, any errors in the calculation results were less than $1 \times 10^{-10} \mathrm{D}$.


Fig. 1. The astigmatic power curve of a cylinder (C) +2.0 diopter (D) $\times 0^{\circ}$ corneal astigmatism

Table 1. Residual astigmatism when a corneal astigmatism of $\mathrm{C}+2.0 \mathrm{D} \times 0^{\circ}$ is corrected by a toric intraocular lens of C-2.5 D rotated $15^{\circ}$

| $\theta$ (Angle from the horizontal axis) (degree) | CA (Corneal astigmatism) $\mathrm{C}+2.0 \mathrm{D} \times 0^{\circ}=2$ $\boldsymbol{\operatorname { s i n }}^{2} \theta$ (diopter) | $\begin{aligned} & \text { TI (125\% (C-2.5D) } \\ & \text { IOL rotated 15 } \\ & \text { C-2.5D } \times 15^{\circ} \\ & =-2.5 \sin ^{\circ}(\theta-15) \\ & \text { (diopter) } \end{aligned}$ | CA+TI (Residual astigmatism) $\begin{aligned} & C+2.0 \times 0^{\circ}+C-2.5 D \times 15^{\circ}=2 \\ & \sin ^{2} \theta-2.5 \sin ^{2}(\theta-15)(\text { diopter }) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 0 | 0.000 | -0.167 | -0.167 |
| 15 | 0.134 | 0.000 | 0.134 |
| 30 | 0.500 | - 0.167 | 0.333 |
| 45 | 1.000 | - 0.625 | 0.375 |
| 60 | 1.500 | -1.250 | 0.250 |
| 75 | 1.866 | - 1.875 | - 0.009 |
| 90 | 2.000 | - 2.333 | - 0.333 |
| 105 | 1.866 | -2.500 | - 0.634 |
| 120 | 1.500 | -2.333 | - 0.833 |
| 135 | 1.000 | -1.875 | -0.875 |
| 150 | 0.500 | -1.250 | -0.750 |
| 165 | 0.134 | -0.625 | - 0.491 |
| 180 | 0.000 | -0.167 | -0.167 |

The residual astigmatic power is represented by the amplitude of the sine wave, the axes of which are the peak and trough of the sine wave, respectively (cf. Fig. 2).

The approximate residual astigmatic axes at $45^{\circ}$ and $135^{\circ}$ are shown in square.
The approximate residual astigmatic power=0.375D-(-0.875D)=1.250D. ("0.375") and ("-0.875") are shown in square.

In (Table 1), column 2 indicates the values for a $\mathrm{C}+2.0 \mathrm{D} \times 0^{\circ}$ corneal astigmatism in $15^{\circ}$ increments, while column 3 indicates the values for a $125 \%$ (C-2.5D) toric IOL in the same increments when rotated $15^{\circ}$. In column 4, the residual astigmatism is calculated for a $\mathrm{C}+2.0 \mathrm{D} \times 0^{\circ}$ corneal astigmatism (column 2 ) $+125 \%$ (C-2.5D) toric IOL when rotated $15^{\circ}$ (column 3). We then created (Fig. 2) from (Table 1).


Fig. 2. The residual astigmatism observed when a cylinder (C) +2.0 diopter (D) $\times 0^{\circ}$ corneal astigmatism is corrected by a C-2.5D toric intraocular lens (IOL) rotated $15^{\circ}$ (cf. Table 1)

CA represents a cylinder (C) +2.0 diopter (D) $\times 0^{\circ}$ corneal astigmatism, TI represents a C$2.5 \mathrm{D} \times 15^{\circ}$ toric IOL, and $\mathrm{CA}+\mathrm{TI}$ represents the residual astigmatism. The approximate residual astigmatic power is $1.250 \mathrm{D}=0.375 \mathrm{D}-(-0.875 \mathrm{D})$, which is derived from (Table 1). Area A is magnified in (Fig. 3).

Next, we demonstrate how to calculate the precise residual astigmatic power and axes. The residual astigmatic power is represented by the amplitude of the sine wave, while the residual astigmatic axes are the peak and the trough of the sine wave, respectively. Therefore, in (Table 1 and Fig. 2), the approximate residual astigmatic power is 1.250 $(=0.375-(-0.875)) \mathrm{D}$, and the approximate residual astigmatic axes are $45^{\circ}$ and $135^{\circ}$. However, more precise calculations can be performed by calculating in $1^{\circ}$ increments. Therefore, we created (Table 2 and Fig. 3) from area a in (Fig. 2). (Table 2 and Fig. 3) reveal thata more precise value for the residual astigmatic power is $1.2608(=0.3804-(-0.8804)) \mathrm{D}$, and more precise residual astigmatic axes are $41^{\circ}$ and $131^{\circ}$.

More precise values for residual astigmatic axes at $41^{\circ}$ and $131^{\circ}$ are shown in square. More precise values for residual astigmatic power=0.3804-(-0.8804)=1.2608D. ("0.3804") and (" -0.8804 ") are shown in square.

Table 2. More precise values for residual astigmatic axes and powers (cf. Fig. 3)

| $\theta$ (Angle from the horizontal axis) (degree) | $\begin{aligned} & \hline \text { CA (Corneal } \\ & \text { astigmatism) } \\ & C+2.0 D \times 0^{\circ} \\ & =2 \sin ^{2} \theta \text { (diopter) } \end{aligned}$ | $\begin{aligned} & \mathrm{TI}(125 \%(\mathrm{C}-2.5 \mathrm{D}) \mathrm{IOL} \\ & \text { rotated } \left.15^{\circ}\right) \\ & \mathrm{C}-2.5 \mathrm{D} \times 15^{\circ}=-2.5 \sin ^{2} \\ & (\theta-15)(\text { diopter }) \end{aligned}$ | CA+TI (Residual astigmatism) $\begin{aligned} & \mathrm{C}+2.0 \times 0^{\circ}+\mathrm{C}-2.5 \mathrm{D} \times 15^{\circ} \\ & =2 \sin ^{2} \theta-2.5 \sin ^{2}(\theta-15) \\ & \text { (diopter) } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 39 | 0.7921 | - 0.4136 | 0.3785 |
| 40 | 0.8264 | - 0.4465 | 0.3799 |
| 41 | 0.8608 | - 0.4804 | 0.3804 |
| 42 | 0.8955 | - 0.5153 | 0.3802 |
| 43 | 0.9302 | - 0.5510 | 0.3792 |
| 44 | 0.9651 | - 0.5876 | 0.3775 |
| 45 | 1.0000 | - 0.6250 | 0.3750 |
| 131 | 1.1392 | - 2.0196 | -0.8804 |
| $C=$ cylinder. $D=$ diopter. IOL=intraocular lens. $\theta=$ angle from the horizontal axis. $C A=C+2.0 D \times 0^{\circ}$ corneal astigmatism. $T I=125 \%$ ( $C-2.5 D$ ) toric $I O L$ rotated $15^{\circ}$. CA $+T I=$ residual astigmatism |  |  |  |



Fig. 3. Magnified area a (cf. Fig. 2)
CA represents a cylinder (C) +2.0 diopter $(\mathrm{D}) \times 0^{\circ}$ corneal astigmatism, TI represents a C $2.5 \mathrm{D} \times 15^{\circ}$ toric IOL, and CA+TI represents the residual astigmatism. The peak value of the residual astigmatism is 0.3804 D at $41^{\circ}$.

Furthermore, we are able to calculate almost any corneal astigmatism and any toric IOL through percentage expressions, because the residual astigmatic power is proportional to the corneal astigmatic power. Similar to the aforementioned calculations, we calculated the residual astigmatism of each angle ( $\theta$ ) from $0^{\circ}$ to $180^{\circ}$ for each IOL ( $75 \%, 80 \%, 90 \%, 100 \%$, and $125 \%$ ) and each $\operatorname{IOL}$ rotation ( $0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}, 25^{\circ}$, and $30^{\circ}$ ). From these calculations, we created (Table 3 and Fig. 4). In addition, we calculated the residual astigmatic axes for each IOL ( $50 \%, 75 \%, 100 \%, 125 \%$, and $150 \%$ ) and each angle rotation $\left(0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}, 25^{\circ}\right.$, and $30^{\circ}$ ), and obtained (Fig. 5). We are unaware of previous reports that contain graphs similar to (Fig. 4 and Fig. 5).(Fig. 5) is useful when considering oblique (OB) astigmatisms.

Table 3. Percentages of the residual astigmatic powers for each intraocular lens rotation

| Angle of IOL rotation (degree) | $\begin{aligned} & 75 \% \text { IOL } \\ & \text { (percentages) } \end{aligned}$ | 80\% IOL (percentages) | $\begin{aligned} & 90 \% \text { IOL } \\ & \text { (percentages) } \end{aligned}$ | 100\% IOL (percentages) | 125\% IOL (percentages) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 25.00 | 20.00 | 10.00 | 0.00 | 25.00 |
| 5 | 29.20 | 25.36 | 19.32 | 17.44 | 31.70 |
| 10 | 39.10 | 36.94 | 34.42 | 34.72 | 46.18 |
| 15 | 51.32 | 50.44 | 50.12 | 51.76 | 63.04 |
| 20 | 64.30 | 64.36 | 65.66 | 68.40 | 80.44 |
| 25 | 77.36 | 78.20 | 80.80 | 84.52 | 97.74 |
| 30 | 90.14 | 91.64 | 95.38 | 100.00 | 114.56 |

Each value (from row 1 to row 7 ) is represented in percentage terms compared to the corneal astigmatic power. Minimum values for each angle are shown in square.

This graph was created from (Table 3). The cylinder power of an $n \%$ IOL is $n / 100 \times$ the corneal astigmatic power ( $n=75,100$, and 125). Point B is the intersection of a $100 \%$ IOL and a $75 \%$ IOL (between $14^{\circ}$ and $15^{\circ}$ ). The corneal astigmatic power is equal to $100 \%$ residual astigmatic power.

## 3. RESULTS

In (Table 3), the residual astigmatic powers produced by each toric IOL rotation can be compared. Namely, the rank orders of each IOL are $100 \%<90 \%<80 \%<75 \%<125 \%$ at $5^{\circ}$ of rotation, $90 \%<100 \%<80 \%<75 \%<125 \%$ at $10^{\circ}$ of rotation, and $90 \%<80 \%<75 \%<100 \%$ $<125 \%$ at $15^{\circ}$ of rotation. Therefore, $100 \%$ IOLs are the best selection for rotations within $5^{\circ}$, while $90 \%$ to $100 \%$ IOLs are best for rotations from $5^{\circ}$ to $10^{\circ}$, and $80 \%$ to $90 \%$ IOLs are best for rotations over $10^{\circ}$. Moreover, (Fig. 4) reveals that the residual astigmatic powers of $125 \%$ IOLs are always higher than those of $100 \%$ IOLs. In contrast, those of $75 \%$ IOLs become lower than those of $100 \%$ IOLs when the IOLs rotation exceeds the point of intersection (point B: between $14^{\circ}$ and $15^{\circ}$ ). Accordingly, the curves of the residual astigmatic powers of lower cylinder power toric IOLs have gentler slopes than those of higher cylinder power toric IOLs. This means that low cylinder power IOLs are a better choice than high cylinder power IOLswhen rotated.


Angle of IOL rotation
Fig. 4. The residual astigmatic powers of $\mathbf{7 5 \%}, \mathbf{1 0 0 \%}$, and $\mathbf{1 2 5 \%}$ intraocular lens (IOL) rotations

In (Fig. 5), corrections with lower cylinder power IOLs (50\%) result in the residual astigmatic axis being close to the corneal astigmatic axis, while corrections with higher cylinder power IOLs $(150 \%)$ result in it being close to the axis of the IOLs. In addition, it is demonstrated that IOLs closer to $100 \%$ produce more oblique (OB) astigmatism. In general, OB astigmatism is not very significant if its power is small; however, the power of $O B$ astigmatism should be minimized [11]. Therefore, low cylinder power toric IOLs are also better from this viewpoint, when exact IOLs are not available for the corneal astigmatism.

The cylinder power of an $n \% I O L$ is $n / 100 \times$ the corneal astigmatic power ( $n=50,75,100,125$, and 150 ). In a $100 \%$ IOL (a), there is no axis when there is no rotation; however, the axis is approximately $45^{\circ}$ when rotated by even $0.1^{\circ}$.


Fig. 5. The residual astigmatic axes of each intraocular lens (IOL) rotation

## 4. DISCUSSION

The advantage of our method is that the table (Table 3) and chart (Fig. 4) prepared beforehand are useful references that allow us to select the most appropriate toric IOLs. This method was facilitated by expressing the cylinder power of toric IOLs in percentage terms compared to the corneal astigmatic power and by using Excel tables; therefore, the calculations are simple. Consequently, (Table 3 and Fig. 4) indicate that a low cylinder power IOL would be a better choice than a high cylinder power IOL when rotated. This opinion is also shared by Felipe et al. [9].On the other hand, some methods have recommended high cylinder power IOLs, contrary to our opinion [12,13]. However, we must remember that slightly higher cylinder power IOLs will cause OB astigmatisms instead of becoming against-the-rule (ATR) astigmatisms when correcting with-the-rule (WTR) astigmatism (Fig. 5). Furthermore, the residual astigmatic powers corrected by high cylinder power IOLs are larger than those corrected by low cylinder power IOLs (Fig. 4). Kobayashi et al. reported that eyes with OB astigmatism had significantly lower visual performance than eyes with WTR or ATR astigmatism [11]. Moreover, the influence of IOL rotation becomes more severe with greater corneal astigmatism and when higher cylinder power IOLs are selected. In fact, one report suggested that the error was particularly relevant in patients in whom higher cylinder power IOLs were implanted [2]. Consequently, we recommend low cylinder power IOLs when correcting corneal astigmatisms.

Clinically, ATR astigmatic change in corneal astigmatisms with advancing age is a documented fact $[14,15]$. Therefore, ATR astigmatism calculations are important. However, we have only calculated WTR astigmatisms in order to make the charts easy to understand.

This is because the differences in calculation results between the two astigmatisms are only the residual astigmatic axes. Thus, the residual astigmatic powers are the same for the two types of astigmatisms. For this reason, (Fig. 4) can also be applied to ATR astigmatisms.

In addition, (Fig. 4) can be used for both clockwise and counterclockwise IOL rotations. While we calculated only counterclockwise rotations, we could similarly calculate the clockwise rotation by changing the two formulas $C-2.5 D \times 15^{\circ}$ and $-2.5 \sin ^{2}(\theta-15)$ in (Table 1) to $C-2.5 \mathrm{D} \times\left(-15^{\circ}\right)$ and $-2.5 \sin ^{2}(\theta+15)$, respectively.

In our report, we did not mention surgically induced astigmatism (SIA) [16,17]. However, SIA does not contradict our method, because our method can also be used to calculate total astigmatisms. From the viewpoint of SIA, there is an opinion that recommends higher cylinder power IOLs [13]. In contrast, we recommend lower cylinder power IOLs, even if we consider including SIA, because slightly higher cylinder power IOLs cause OB astigmatism (Fig. 5).

To the best of our knowledge, there have been no methods for calculating residual astigmatisms using Excel spreadsheets, $\sin ^{2} \theta$, and percentage expressions of cylinder power IOLs compared to the corneal astigmatic power.

## 5. CONCLUSION

In conclusion, we were able to obtain precise and visually comprehensible charts that are useful for selecting toric IOLs, particularly when exact IOLs are not available.

## CONSENT

Not applicable.

## ETHICAL APPROVAL

Not applicable.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Peer-review history:
The peer review history for this paper can be accessed here: http://www.sciencedomain.org/review-history.php?iid=523\&id=23\&aid=5333


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