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Three-dimensional MHD Mixed Convection Casson Fluid Flow over an Exponential Stretching Sheet with the Effect of Heat Generation

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Authors' contributions

This work was carried out in collaboration between both authors. Author KS designed the study, wrote the protocol and supervised the work. Authors KS and BS carried out all laboratories work and performed the statistical analysis. Author KS managed the analyses of the study. Author BS wrote the first draft of the manuscript. Author KS managed the literature searches and edited the manuscript. Both authors read and approved the final manuscript.

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Abstract

In this article, the governing equation which models the problem of steady three-dimensional Casson fluid flow over an exponentially stretching surface in the presence of Lorentz force is investigated. We have considered the effects of heat generation and mixed convection. Similarity transformations are used to convert the partial differential equations to set of ordinary differential equations. These equations are solved by applying Keller Box method. The effects of Magnetic parameter, mixed convection parameter, heat source/sink, Casson parameter, ratio parameter are investigated on the velocity and temperature profiles graphically.

Keywords: Magnetohydrodynamic (MHD); Casson fluid; heat generation.

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1 Introduction

The study of MHD, boundary layer flow has many applications in industrial fields. Magnetohydrodynamic is the study of Magnetic properties of electrically conducting fluids. Some examples are plasma, liquid metals and salt water. The field of MHD was discussed by Hannes Alfven [1]. Magnetic fields play a key role in star formation. Ali et al. [2] discussed MHD boundary layer flow and heat transfer over a stretching sheet with induced magnetic field. M. Turkyilmazoglu [3] and Kartini Ahmad et al. [4] explained three dimensional MHD flow and heat transfer over a stretching sheet with various physical effects. Crane [5] discussed on stretching sheet.

Casson fluid is one type of non Newtonian fluid. Casson fluid can be defined as a shear thinning liquid which is supposed to have an infinite viscosity at zero rate of shear and a yield stress under which no flow occurs and zero viscosity at an infinite rate of shear. If yield stress greater than the shear stress is applied to the fluid, it behaves like solid. If yield stress less than the shear stress then fluid starts move. Honey, soup, concentrated juices are few examples of Casson fluid. Three dimensional Casson fluid flow past a linearly stretching sheet is investigated by Mahantha et al. [6] and Nadeem et al. [7]. Magyari and Keller [8] explained heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. Mixed convection heat transfer on exponential stretching sheet with magnetic field investigated by Pal [9]. Nadeem et al [10] described MHD flow of a Casson fluid on exponential shrinking sheet. Naramgari Sandeepa et al. [11] explained about three dimensional Casson fluid flow. Subbarao et al. [12] investigated the boundary layer flows of non-newtonian fluid with thermal slip. Three dimensional MHD flow of Casson fluid in porous medium with heat generation is described by Shehzad [13]. We have useful applications for different heat source/sink arrangements. Krishnendu Bhattacharyya [14] explained the effect of heat source/sink on MHD flow over a shrinking sheet. F.M Hady [15] described the effects of heat source/sink on MHD viscoelastic fluid. Generalized three dimensional flow due to a stretching surface is explained by Ariel [16]. Liu IC et al. [17] described the flow and heat transfer for three dimensional flow over an exponentially stretching surface. MHD boundary layer flow of Casson fluid with thermal radiation on exponential stretching sheet is derived by Mukhopadhyay et al. [18]. P.K Kameswarn [19] investigated Dual solutions of Casson fluid flow over a stretching sheet. Chamkha [20] described Hydromagnetic three dimensional free convection on a vertical stretching sheet with heat generation. Sulochana et al. [21] described the effect of heat source/sink on three dimensional Casson fluid with soret and thermal radiation. Unsteady three-dimensional flow of casson-carreau fluid is explained by Raju and Sandeep [22]. Similarity solution of three-dimensional Casson nanofluid with convective conditions is explained by Sulochana et al. [23]. Animasaun Isaac Lare [24] explained Casson fluid flow over an exponential stretching sheet with heat generation.

2 Mathematical Formulation

Let us consider a three dimensional steady, laminar, incompressible MHD mixed convection flow of a Casson fluid over an exponential stretching sheet. The sheet z=0 is stretched with the velocities $U_w = U_0 e^{\frac{x+y}{L}}$ and $V_w = V_0 e^{\frac{x+y}{L}}$ along the xy plane as shown in the above Fig. 1. Where U_0, V_0 are constants. The uniform magnetic field is applied in the z-direction that is perpendicular to the flow. A heat source/sink placed within the flow to allow for heat generation or absorption effects.

Consider u, v and w are velocity components in the directions of x, y and z respectively in the flow field. The governing equations of continuity, momentum and energy are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \upsilon(1 + \frac{1}{\beta})\frac{\partial^2 u}{\partial z^2} + g\beta_T(T - T_\infty) - \frac{\sigma B^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\upsilon(1+\frac{1}{\beta})\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B^2}{\rho}v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\frac{Q_1}{\rho C_P}(T-T_\infty) \tag{4}$$

where ρ is the density of the fluid, σ is the electrical conductivity, υ is the kinematic viscosity, $B=B_0e^{\frac{x+y}{2L}}$ is the dimensional magnetic induction, $\alpha(=\frac{K}{\rho C_p})$ is the thermal diffusivity, K is the thermal conductivity, C_p is the specific heat capacity at constant pressure, T_∞ is the free stream temperature, $Q_1=Q_0e^{\frac{x+y}{L}}$ is the dimensional heat generation. The boundary conditions considered are defined as

$$u=U_w,\ v=V_w,\ w=0,\ T=T_w \qquad at\ \ z=0\,. \tag{5}$$

$$u\to 0,\ \ v\to 0,\ T\to T_\infty \quad as\ \ z\to\infty. \tag{6}$$

Where $U_w=U_0e^{\frac{x+y}{L}},\ V_w=V_0e^{\frac{x+y}{L}},\ T_w=T_\infty+T_0e^{\frac{x+y}{2L}}$, T_w is the temperature of the fluid.

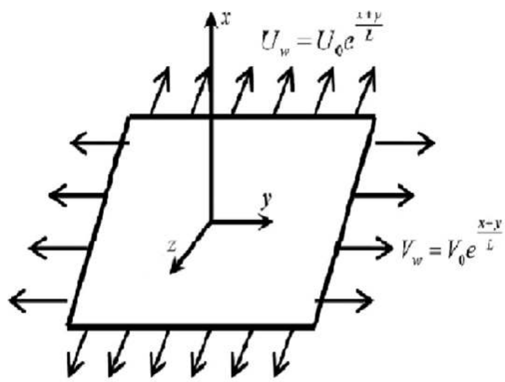

Fig. 1. Physical model and coordinate system

Introducing the similarity transformations

$$\eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x+y}{2L}}z\ ,\ u=U_0e^{\frac{x+y}{L}}f'(\eta)\,,\ v=U_0e^{\frac{x+y}{L}}g'(\eta)\ ,\ T=T_\infty+T_0e^{\frac{x+y}{2L}}\theta(\eta)\,,$$

$w=-\sqrt{\frac{\upsilon U_0}{2L}}\ e^{\frac{x+y}{2L}}(f+\eta f'+g+\eta g')$ in to the equations (2) & (3) & (4) we get

$$(1+\frac{1}{\beta})f'''+(f+g)f''-Mf'+2\lambda\theta=0, \tag{7}$$

$$(1+\frac{1}{\beta})g'''+(f+g)g''-Mg'=0, \tag{8}$$

$$\frac{1}{Pr}\theta''-[(f'+g')\theta+(f+g)\theta']+2Q\theta=0. \tag{9}$$

Where M = $\frac{2\sigma B_0{}^2}{\rho U_0}$ is the magnetic parameter, Pr = $\frac{\nu}{\alpha}$ is the Prandtl number, λ = $\frac{Gr_x}{Re_x{}^2}$ is the mixed convection parameter, Re = $\frac{U_{0}L}{\nu}e^{\frac{x+y}{L}}$ is the Local Reynolds number, $Gr_x=\frac{g\beta_T(T_w-T_\infty)x^3L}{\nu^2}$ is the Local Grashof number, $Q=\frac{2Q_0}{\rho U_0C_p}$ is the local heat source (or sink) parameter.

The boundary conditions are reduced to

$$\eta=0: \ f'(\eta) = 1, g'(\eta) = C, f(\eta) = 0, g(\eta) = 0, \theta(\eta) = 0.$$

$$\eta\to\infty: \ f'(\eta) \to 0, g'(\eta) \to 0, \theta(\eta) \to 0.$$

Here $C = \frac{V_0}{U_0}$ is the ratio parameter. The skin friction coefficient along the directions of x and y are $C_{fx} = \frac{2\tau_w}{\rho U_w^2}$ [$C_{fx} Re_x^{1/2} = (1+\frac{1}{\beta})f''(0)$], $C_{fy} = \frac{2\tau_w}{\rho V_w^2}$ [$(\frac{x}{y}) C_{fy} Re_x^{1/2} = C\left(1+\frac{1}{\beta}\right) g''(0)$] . $Nu_x = \frac{x q_w}{k(T_w - T_\infty)}$ [$\sqrt{2} Nu_x = -(\frac{x}{L}) Re^{1/2} \theta'(0)$] is the local Nusselt number, where $q_w = -K\left(\frac{\partial T}{\partial z}\right)_{z=0}$ is the rate of heat transfer.

3 Methods of Solution

The governing equations with boundary equations are solved numerically by using finite difference scheme known as Keller box method which is described by Cebeci and Bradshaw [25]. This method involves the following steps.

Step1: Reducing higher order ODEs (systems of ODES) in to systems of first order ODEs.
Step2: Writing the systems of first order ODEs into difference equations using central difference scheme.
Step3: Linearizing the difference equations using Newton's method and writing it in vector form.
Step4: Solving the system of equations using block elimination method.

4 Numerical Discussion

In this study to obtain numerical solution we have used Matlab software and the step size we considered as $\Delta\eta = 0.01$. The following Table 1 shows that the comparison between values of the skin friction coefficient by present method and that of Nadeem et al. [26] in the absence of Pr, λ and Q.

5 Results and Discussion

In this paper, we used the values of parameters M=0.5, Pr=0.72, λ=0.1, Q=0.1, β=0.2, C=0.1 for throughout graphical representations unless otherwise mentioned.

Fig. 2 illustrates the influence of magnetic parameter M on the velocities $f'(\eta)$ and $g'(\eta)$. As The value of M increases, velocity decreases due to Lorentz force. Therefore the boundary layer thickness decreases. Fig. 3 shows the effect of Casson parameter β on velocity profiles $f'(\eta)$ and $g'(\eta)$. An increase in β leads to an increase in plastic dynamic viscosity that creates resistance in the fluid flow. Therefore we analyzed that the velocities of the fluid and their boundary layer thicknesses are decreasing functions of β. Fig. 4 explains that the effect of Prandtl number on the temperature profile. The fluids with high Prandtl number have low thermal diffusivity. Increasing the Prandtl number gives rise to a decrease in temperature.

Fig. 5 depicts the effect of velocity ratio parameter C on $f'(\eta)$ and $g'(\eta)$. An increase in ratio parameter decreases the boundary layer thickness for $f'(\eta)$ and increases in $g'(\eta)$. Physically when C increases the stretching rate increases in the y-direction. Thus the velocity increases in the y direction. Her C=0 represents two dimensional case. If C=1 the behaviour of the flow along both the directions is same.

Fig. 6 shows that effect of heat source (or sink) parameter Q on temperature profile. In general heat generation parameter in the fluid increases the temperature. Therefore we have seen that an increase in Q enhances the thermal boundary layer thickness and temperature. Fig. 7 demonstrates that influence of the mixed convection parameter λ on the profile of velocity. It is observed that the velocity profile increases as the value of the λ increases due to buoyancy effect.

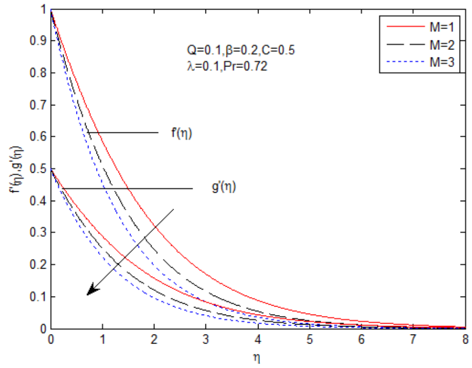

Fig. 2. Velocity profile for various values of M

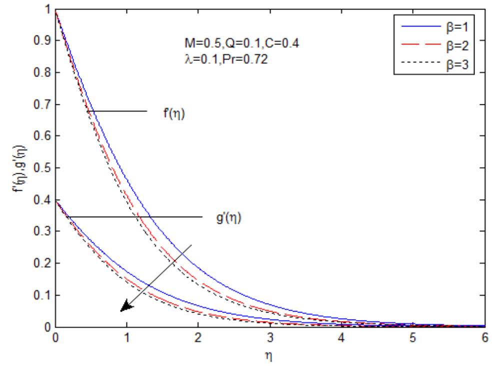

Fig. 3. Velocity profile for various values of β

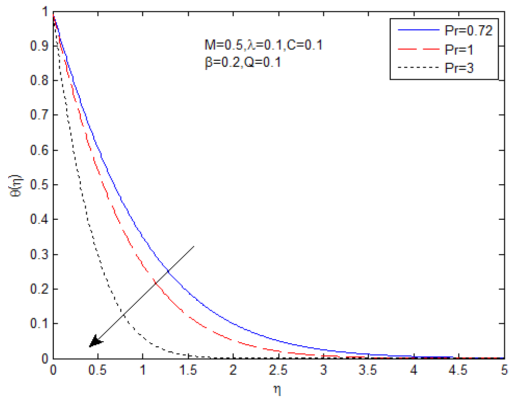

Fig. 4. Temperature profile for various values of Pr

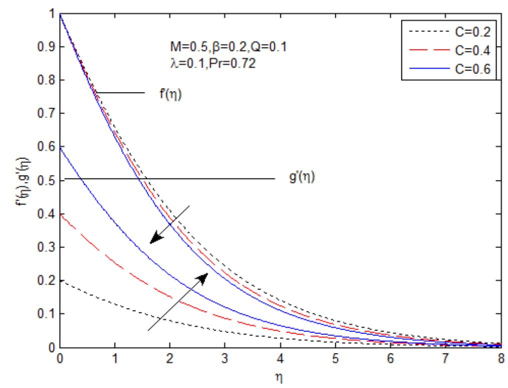

Fig. 5. Velocity profile for various values of C

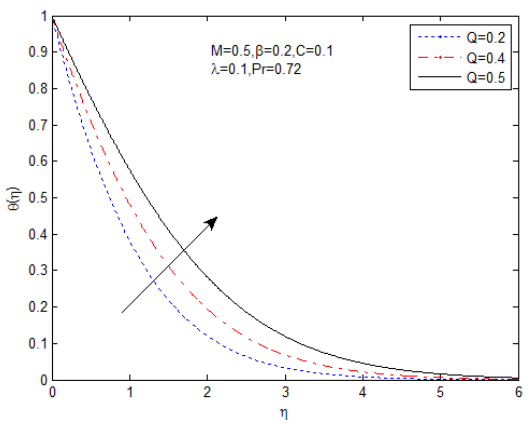

Fig. 6. velocity profile for various values of Q

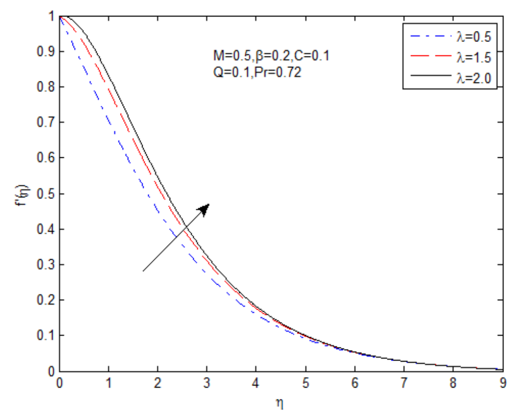

Fig. 7. Velocity profile for various values of λ

Table 1. Comparison of results for $-\left(1+\frac{1}{\beta}\right)f''(0)$ and $-\left(1+\frac{1}{\beta}\right)g''(0)$ with previous available data

M	β	Nadeem et al. (When C=0.5)		Present method (when C=0.5)	
		$-\left(1+\frac{1}{\beta}\right)f''(0)$	$-\left(1+\frac{1}{\beta}\right)g''(0)$	$-\left(1+\frac{1}{\beta}\right)f''(0)$	$-\left(1+\frac{1}{\beta}\right)g''(0)$
10	1	4.7263	2.3276	4.7262	2.3276
	5	3.6610	1.8030	3.6611	1.8032
	∞	3.3420	1.6459	3.3420	1.6459

6 Conclusions

The graphical representations of three-dimensional MHD Casson fluid flow over an exponential stretching sheet were discussed. We have the following observations.

- The velocity increases with increasing values of the magnetic parameter M and Casson parameter β on $f'(\eta)$ and $g'(\eta)$.
- An increase in the Prandtl number Pr decreases the temperature profile.
- With increasing values of the ratio parameter C decreases the velocity profile $f'(\eta)$ and increases the velocity profile $g'(\eta)$.
- Increase in heat source (or sink) parameter increases the temperature.
- The velocity profile increases, as the value of λ increases.

Competing Interests

Authors have declared that no competing interests exist.

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