Journal of Scientific Research & Reports
12(5): 1-8, 2016; Article no.JSRR.30036
ISSN: 2320-0227

SCIENCEDOMAIN *international*
www.sciencedomain.org

Selection of Bayesian Multiple Deferred Sampling Plan (BMDS-1) with Weighted Poisson Model Based on AOQ and ATI Curve

K. Subbiah[1*] and M. Latha[2]

[1]*Department of Statistics, Government Arts College, Udumalpet-642126, Tamil Nadu, India.*
[2]*Kamarajar Government Arts College, Surandai- 627859, Tamil Nadu, India.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JSRR/2016/30036
Editor(s):
(1) Shunpu Zhang, Department of Statistics, University of Nebraska – Lincoln, USA.
Reviewers:
(1) Ming-Chang Lee, National Kaohsiung University of Applied Sciences, Taiwan.
(2) Ismail Olaniyi Muraina, Adeniran Ogunsanya College of Education, Nigeria.
Complete Peer review History: http://www.sciencedomain.org/review-history/17133

Original Research Article

Received 13th October 2016
Accepted 22nd November 2016
Published 5th December 2016

ABSTRACT

Bayesian Acceptance Sampling Approach is associated with utilization of prior process history for the selection of Distributions (viz., Gamma Poisson, Beta Binomial) to describe the random fluctuations involved in Acceptance Sampling. Latha [1] has studied Bayesian Chain Sampling Plan – 1 involving designing of Bayesian Chain Sampling Plan indexed through Acceptance Quality Limit (AQL), Limiting Quality Limit (LQL), Overall Average Outgoing Quality Limit (OAOQL), and Maximum Allowable Average Percent Defective (MAAPD). The main thrust of this paper is to account for the possibility of dependence among the items of a sample. In paper a procedure is developed to draw an Average Outgoing Quality Level (AOQL), Overall Average Outgoing Quality Limit (OAOQL) curve by using Gamma Poisson distribution and compare the different procedures in order to show the ambiguity of the procedure and their results. It provides an investigation into the robustness of single sampling procedure indexed by Acceptance Quality Level (AQL) and Average Outgoing Quality Level (AOQL).

**Corresponding author: E-mail: bilyabraham2008@gmail.com;*

Keywords: Bayesian multiple deferred state – 1 (c1, c2) sampling plan; weighted Poisson distribution with gamma prior; Acceptance Quality Level (AQL); Average Outgoing Quality Level (AOQL); Average Total Inspection (ATI).

1. INTRODUCTION

An acceptance sampling is one of three major statistical areas which are used for quality control and improvement. An acceptance sampling is a form of testing which involves taking random sample of lots and measuring them against pre-determined standards. Depending on kind of production, a lot or batch can contain one or in majority cases, more than one raw material, component of final product.

The lot can be inspected immediately following production or before the product is shipped to the customer. On the other side, the lots can be inspected as they are received from the supplier. In the first case, it is conducted outgoing inspecting and the second case incoming inspection (Montgomery et al. [2]).

If a supplier's process provides no defective units and no economic justification to make an inspection exist, the lot is accepted without inspection. But when the defective units might result in a considerably high failure costs for the buyers, or when it is known that a supplier's process does not meet a certain level of quality standards, then is a 100% inspection recommended [2].

The approach of no checking may be guaranteed when the process capability is known and the probability of defective product is very small. In some cases, incoming materials from various suppliers may not be inspected because the supplier has demonstrated outstanding quality capabilities. When the process capability and the product quality level are not known, no checking usually results in increased costs for reworking defective product. When the risks involved are known, this technique will result in significant savings. When the risks are not known, this technique may cause significant losses and problems to the company.

Sampling involves a risk that the sample may not adequately reflect the condition of the lot. i.e., it may not represent the lot correctly. Sampling risks are of two kinds (i) Producer's risk (ii) Consumer's risk. This study examines single sampling plans for variables indexed by AQL and AOQL under measurement error. Procedures and tables are provided for selecting single sampling plans for variables for given AQL and AOQL when rejected lots are 100% inspected for replacement of nonconforming units. For a particular sampling plan in operation for observed measurement, a method of determining the AOQ curves is described for various errors sizes. Model description for Variable Single Sampling Plan indexed by AQL and AOQL under Measurement Errors. The concept of the Multiple Dependent (or deferred) State (MDS) sampling plan was introduced by Wortham & Baker [3]. MDS Sampling Plan belong to the group of conditional sampling procedures. In these producers, acceptance or rejection of a lot is based not only on the sample from that lot, but also on sample result from past or future lots. Latha and Subbiah [4] have given the procedure for the selection of multiple deferred state sampling (MDS – 1) plan through the weighted Poisson model with gamma prior. Soundararajan and Vijayaraghavan [5] extended this approach to multiple deferred sampling plan of type MDS-1(0,2) limiting to the acceptance number at 0 and 2. Subramani and Govindaraju [6] have presented tables of the selection of multiple deferred state MDS – 1 sampling plan for given acceptable and limiting Quality using Poisson distribution. This paper gives tables and procedure for selecting the multiple deferred state MDS -1 sampling plan of Varest [7] involving operating characteristic curve (OC) using weighted Poisson distribution with Gamma prior. Suresh [8] have presented tables of Bayesian chain sampling plan and our present work is purely based on Bayesian Multiple Deferred Sampling (BMDS – 1) plan.

1.1 Bayesian Acceptance Sampling

Bayesian Acceptance Sampling Approach is associated with utilization of prior process history for the selection of distributions (viz., Gamma Poisson, Beta Binomial) to describe the random fluctuations involved in Acceptance Sampling. Bayesian sampling plans requires the user to specify explicitly the distribution of defectives from lot -to lot. The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution, and the empirical knowledge based on the sample leads to the decision on the lot.

1.2 MDS – 1 Plan

The MDS – 1 plan is applicable to the case of Type B situations where lots expected to be of the same quality are submitted for inspection seriously in the lot production.MDS – 1 plans are extensions of chain sampling plans of Dodge's [9] type ChSP – 1. Both the MDS – 1 and chain sampling plans achieve a similar reduction in sample size when compared to the unconditional plans, such as single and double sampling plans. The Operating procedure of the MDS – 1 plan as given by.

(1) From each submitted lot, select a sample of n units and test each unit for conformance to the specified requirements.
(2) Accept the lot if x, the observed number of nonconformities, is less than or equal to c_1; reject the lot if x is greater than c_2.
(3) If $c_1 < x < c_2$ accept the lot, provided in each of the sample taken from the preceding or succeeding m lots, the number of nonconformities found is less than or equal to c_1. The lot otherwise rejected.

1.3 Weighted Poisson Distribution

Rao CR [10] introduced the concept of weighted distribution when the samples are recorded without a sampling frame that enables random samples to be drawn. The weight function that usually appears in the scientific and statistical literature is ω(X) =X, which provides the size biased version of the random variable. The size – biased version of order k, which corresponds to the weight ω (X) $=X^k$, for k any real positive number has also been widely used. Joan Del Castillo and Peres- Casany [11] applied the weighted Poisson distribution that results from the modification of the Poisson distribution with the weight ω(X) $=X^k$ can also considered as a mixture of the size biased version of the Poisson distribution. Patil GP, Rao CR and Ratnaparki MV [12] have proved that given a random variable X, the weighted version X^k is stochastically greater or smaller than the original random variable X according as the weight function ω(X) is monotonically increasing or decreasing to X. Patil GP, Rao CR and Ratnaparki MV [13] pointed out that the importance of the size-biased version of a random variable X. They show that many classical discrete distributions have a size-biased version of the same form with the variable reduced by unity.

In the construction of acceptance sampling plan, size- biased version of random variable about defectives play an important role. The weighted distributions are more suitable distributions than the classical distributions like Binomial, Poisson and Negative Binomial. The weighted Poisson distribution plays an important role in acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned different weights based on its importance or usage. The probability mass function of weighted Poisson distribution is given by:

$$p(x,n,p,k) = \frac{x^k p(x)}{\Sigma x^k p(x)} \quad x = 1,2,3 \tag{1}$$

Where

$$p(x) = \frac{e^{-np}(np)^x}{x!} \quad x = 1,2,3 \tag{2}$$

Here X^k is the corresponding weight for each outcome and 'k' is a constant. The Poisson distribution can be seen as the particular case of the weighted Poisson distribution when k = 0. The probability mass function of the weighted Poisson distribution for k = 1 is

$$p(x) = \frac{e^{-np}(np)^{x-1}}{(x-1)!} \quad x = 1,2,3 \tag{3}$$

When p follows gamma prior distribution with density function

$$(p) = \frac{e^{-pt} t^s p^{s-1}}{\Gamma s}, \ p > 0, \ s, t, > 0 \tag{4}$$

Where s and t are the parameters and the mean value of distribution $= \frac{s}{t}$.

2. AVERAGE OUTGOING QUALITY LIMIT (AOQL)

The second fundamental curve in the description of a sampling plan is the average outgoing quality (AOQ) curve. This is another indication of the protection afforded by the sampling plan. Naturally the average quality, after the sampling inspection and sorting of rejected lots, depends upon the lot quality being offered. Hence, we shall for the present assume that lots of but one quality p' are offered. Then the outgoing quality will depend upon p'.

There are several interpretations possible in the meaning of AOQ (refer Irving W. Burr – [14]). In practice, they give about the same results, and hence the easiest method is commonly used.

The differences depend upon how the defectives are treated. We have the following:

1. The defectives found in the samples may be retained with the accepted lots and not replaced by good pieces. This is the simplest assumptions.
2. The defectives found in the samples and sorting may all be removed, but not replaced by good pieces. This is the usual assumption where sampling inspection is done at same considerable distance from the places when the pieces were produced. Hence it is not feasible to replace the defectives.
3. The defectives found in the samples and sorting may all be removed and replaced by good pieces. This is the usual assumption when sampling inspection is done close to the place where the pieces are produced, so that the defectives may be replaced. This is typical of final inspection.

In all three cases, as we do throughout this paper, we assume that the inspectors find all the defectives, and only the defectives in the samples and lots. Knowing the value of the AOQL for a sampling plan is very helpful, because it tells, what is the worst long –run average quality which will leave the inspection department.

The AOQ curve pictures this relationship between incoming quality and outgoing quality .It will be noticed that the curves show the fraction defective of the outgoing material and therefore fluctuations inversely with quality. The maximum ordinates of the curve measures the worst possible quality of material turned out by the plan and is known as the Average Outgoing Quality Limit or briefly the AOQL. It is to be noted once again that this is an average value over many lots. The AOQL is the maximum of the AOQs for all possible incoming qualities for a given acceptance sampling plan (refer Duncan - [15]).

The use of the AOQ Curve and the concept of the AOQL assume that it is feasible to sort a lot 100 percent for defectives. If this is not feasible or is impossible, for example, in destructive tests, then these features of a sampling plan are meaningless.

2.1 Mathematical Model

2.1.1 Bayesian MDS-1 plan

Based on Hald [16], APA functions for MDS-1 Plan with Gamma Poisson distribution is obtained as

$$\overline{P} = \overline{P}c_1+[\overline{P}c_2-\overline{P}c1][\overline{P}c_1]^m \tag{5}$$

Where

$$\overline{P}c_1 = \sum_{x=1}^{c1} \frac{1}{\beta(x,s-1)}\frac{s^s}{(s-1)}\frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}\ ; x = 1,2,3,\ldots$$

$$\overline{P}c_2 = \sum_{x=1}^{c2} \frac{1}{\beta(x,s-1)}\frac{s^s}{(s-1)}\frac{\mu^{x-1}}{(\mu+s)^{x+s-1}}\ ; x = 1,2,3,\ldots$$

The average probability of acceptance is given by

$$\bar{P} = \frac{s^s}{(s+n\mu)^s} + \frac{s^{ms+s+1}\,n\mu}{(s+n\mu)^{ms+s+1}} \tag{6}$$

The Overall Average Outgoing Quality (OAOQ) for a Bayesian sampling plan is given by

$$OAOQ = \int pP\text{a}(p)\text{w}(p)\,dp \tag{7}$$

Where W(p) is the distribution function of the product quality p, OAOQL is maximum value of OAOQ. The expression in the right hand side is equated to the required probability of acceptance and then nμ values are obtained.

For (BMDSP-1),

$$nOAOQ = \frac{s^{s+1}n\mu}{(s+n\mu)^{s+1}} + \frac{(s+1)s^{ms+s+m+1}(n\mu)^{m+2}}{(s+n\mu)^{ms+s+m+2}} \tag{8}$$

Differentiating equation (6) with respect to μ and equating to which results with

The below Equation is solved for the values of nμ for different values of s and m .Substituting nμ in equation (6) OAOQL values are obtained

$$\frac{s^{s+1}}{(s+n\mu)^{s+1}} - \frac{s^{s+1}n\mu(s+1)}{(s+n\mu)^{s+2}} - \frac{s^{m+s+m*s+1}n\mu^{m+2}(s+1)(m+s+ms+2)}{(s+n\mu)^{m+s+ms+3}} + \frac{s^{m+s+ms+1}n\mu^{m+1}(m+2)(s+1)}{(s+n\mu)^{m+s+ms+2}} = 0 \tag{9}$$

Table 1. OAOQ values of BMDS-1 Plan for m=1

nμ\s	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.02	0.01942	0.01949	0.01952	0.01954	0.01955	0.01956	0.01956	0.01957
0.03	0.02873	0.02886	0.02893	0.02897	0.029	0.02902	0.02903	0.02905
0.04	0.03778	0.03801	0.03813	0.0382	0.03825	0.03828	0.03831	0.03833
0.5	0.29532	0.31151	0.32023	0.32569	0.32942	0.33213	0.03342	0.3358
0.6	0.32473	0.34517	0.35634	0.36338	0.36822	0.37175	0.37446	0.37656
0.7	0.34747	0.3718	0.38528	0.39385	0.39977	0.40411	0.40742	0.41004
0.8	0.3644	0.3921	0.40764	0.41758	0.42449	0.42957	0.43346	0.43654
0.9	0.37636	0.40677	0.42401	0.43512	0.44287	0.44858	0.45297	0.45644
1	0.38409	0.41652	0.43505	0.44705	0.45546	0.46168	0.46665	0.47026
1.5	0.3806	0.41335	0.43221	0.44448	0.45311	0.45951	0.46445	0.46837
2	0.34375	0.3667	0.37898	0.38657	0.39169	0.39537	0.39814	0.40029
2.5	0.29976	0.31034	0.31425	0.31582	0.31643	0.31659	0.31655	0.3164
3	0.25836	0.25781	0.25437	0.25076	0.24752	0.24473	0.2434	0.24029
4	0.19204	0.17657	0.16406	0.15449	0.14708	0.14124	0.13653	0.13266

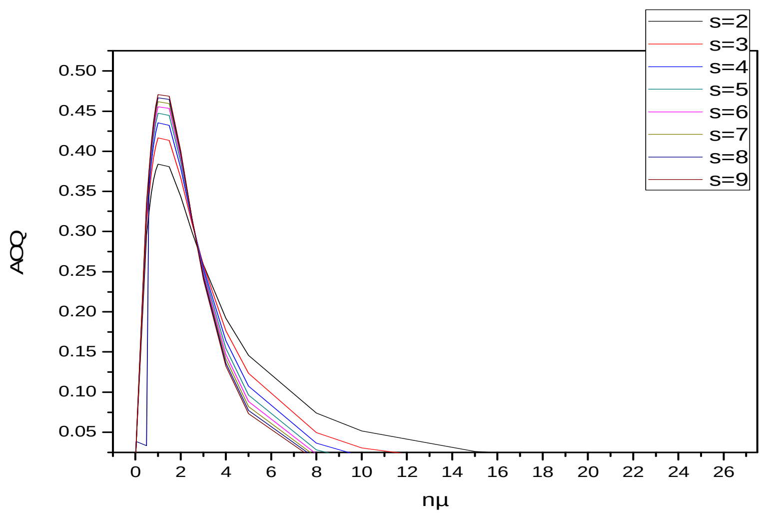

Fig. 1. OAOQ curves of BMDS-1 Plan for m=1

Table 2. OAOQ values of BMDS-1 Plan for m=2

nμ\s	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.02	0.01941	0.01948	0.01951	0.01953	0.01954	0.01955	0.01956	0.01956
0.03	0.02869	0.02883	0.0289	0.02894	0.02397	0.02399	0.02901	0.02902
0.04	0.0377	0.03794	0.03806	0.03814	0.03882	0.03822	0.03825	0.03827
0.5	0.26607	0.28112	0.28933	0.2945	0.29805	0.30064	0.30262	0.30417
0.6	0.2872	0.3055	0.31562	0.32204	0.32647	0.32972	0.3322	0.33415
0.7	0.30242	0.32349	0.33528	0.34281	0.34804	0.35188	0.35482	0.35715
0.8	0.31279	0.33605	0.34919	0.35765	0.36355	0.3679	0.37123	0.37387
0.9	0.31917	0.344	0.35815	0.3673	0.37371	0.37844	0.38208	0.38497
1	0.32231	0.34808	0.36286	0.37246	0.37919	0.38418	0.38418	0.38802
1.5	0.30807	0.33098	0.34396	0.35232	0.35815	0.36246	0.36576	0.36838
2	0.27344	0.28706	0.29382	0.29775	0.30027	0.302	0.30325	0.3042
2.5	0.2371	0.241	0.24129	0.24062	0.23973	0.23882	0.23798	0.23722
3	0.20474	0.20068	0.19587	0.19168	0.18821	0.18536	0.18299	0.18099
4	0.15465	0.14056	0.12988	0.12194	0.11591	0.11119	0.10743	0.10436

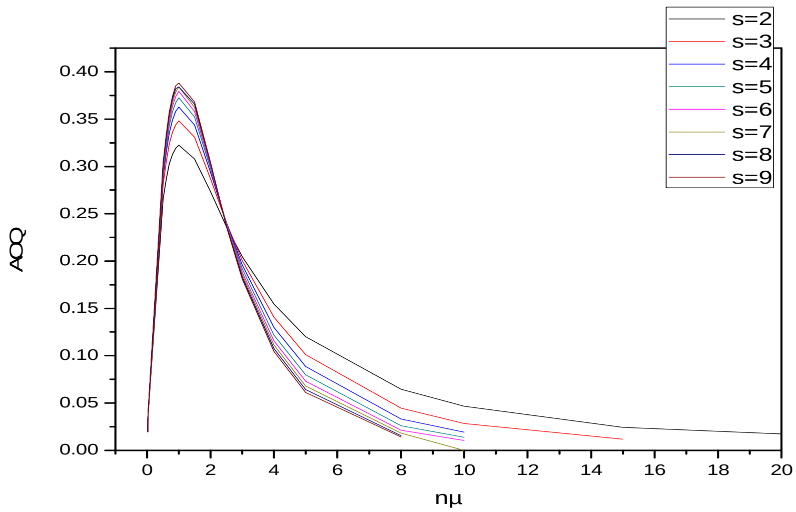

Fig. 2. OAOQ curves of BMDS-1 Plan for m=2

Table 3. ATI values of BMDS-1 Plan for m=1

n/s	2	3	4	5	6	7	8	9
2	465.625	483.643	494.146	501.076	506.008	509.698	512.569	514.873
3	628.462	666.019	688.537	703.603	714.412	722.548	728.902	734.005
4	732.106	778.051	804.889	822.466	834.859	844.057	851.149	856.783
5	799.282	846.685	873.226	890.029	901.567	909.937	916.273	921.232
8	899.218	938.17	957.025	967.645	974.26	978.679	981.811	984.106
10	929.476	961.822	976.087	983.512	987.841	990.577	992.422	993.709
15	964.45	985.141	992.566	995.797	997.399	998.281	998.803	999.127
25	985.726	995.906	998.551	999.406	999.73	999.865	999.9242	999.9562

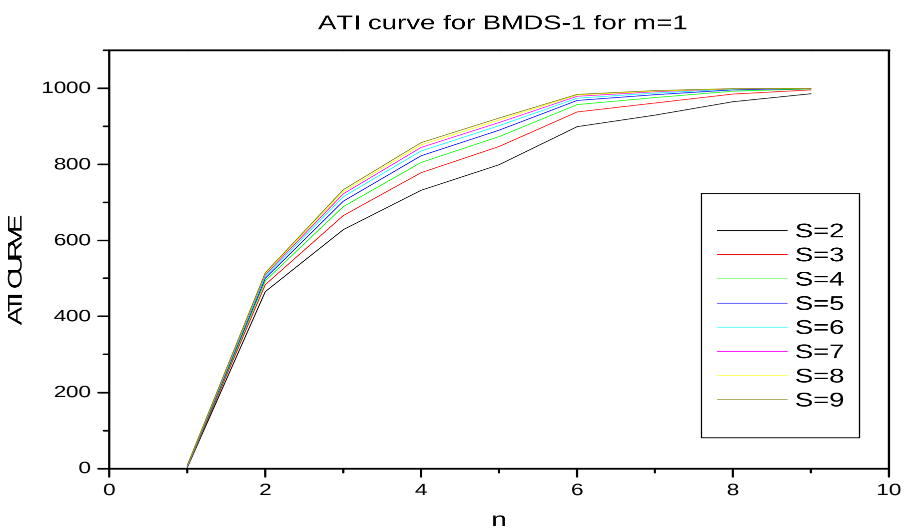

Fig. 3. ATI curves of BMDS-1 Plan for m=1

Table 4. ATI values of BMDS-1 Plan for m=2

n/s	2	3	4	5	6	7	8	9
2	507.817	530.182	543.223	551.827	557.929	562.501	566.047	568.882
3	663.931	702.271	724.78	739.63	750.178	758.062	764.182	769.06
4	757.711	801.37	826.309	842.428	853.687	861.994	868.366	873.415
5	817.156	860.995	885.133	900.307	910.666	918.784	923.842	928.279
8	905.653	941.671	959.158	969.067	975.277	979.453	982.414	984.601
10	933.049	963.352	976.852	983.944	988.111	990.757	992.539	993.808
15	965.521	985.42	992.656	995.833	997.417	998.29	998.803	999.127
25	985.915	995.932	998.551	999.406	999.73	999.865	999.9242	999.9563

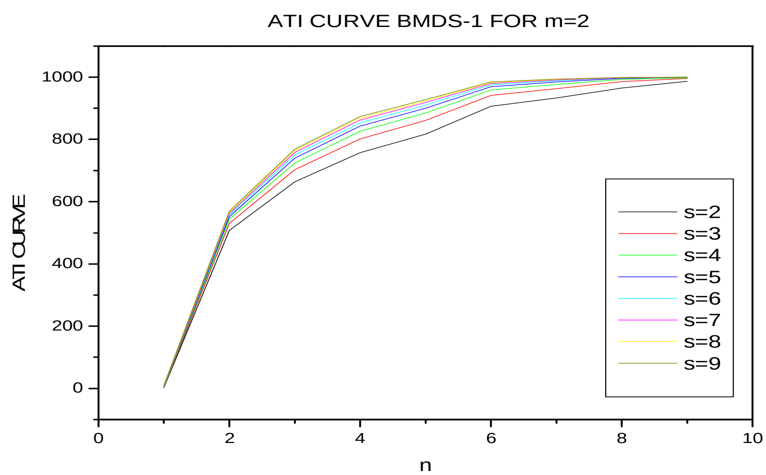

Fig. 4. ATI curves of BMDS-1 Plan for m=2

2.1.2 Analysis of AOQ values

From the Tables 1 and 2 it is observed that, AOQ values corresponding to various values of s and other fixed values of m parameters. It is observed that for a given set of values, when s increases the AOQ values decrease at all levels of µ. Further comparing with the conventional sampling plan, the AOQL values decreases at all levels of p. This ensures protection to the consumer against unsatisfactory lot quality, where on the interest of the producers against satisfactory quality level will be safeguarded. When s values are small from the tables for small values of m, it gives less protection to the consumer and more protection to the producers. When m is increased, the corresponding average outgoing quality is very small at all levels of lot quality µ and the consumer will be safeguarded.

From the Figs. 1 and 2 it is observed that for small values of m, moderate protection to the consumer is extended, while mere is significant effect on the protection to the producer when m = 1, as it provides more chance of accepting the lot of satisfactory quality. But as m is increased the AOQ values are decreased at the region of interest of the producers while they coincide with each other at the lower portion. It is quite evidence from the figure that the AOQ curves converge towards the AOQ curve corresponding to the largest values of m. Further, from the figure, it is observed that large values of m give more protection to the consumer.

2.1.3 Average total inspection

It is sometimes necessary to determine the average amount of inspection per lot in the application of such rectification schemes, including 100 percent inspection of rejected lots. This average, called the average total inspection (ATI), is made up of the sample size n on every lot plus the remaining (N-n) units on the rejected lots (refer Irving W. Burr – [14])., so that

$$ATI= n+(N-n)(1-\bar{P})$$

Consider the sampling plan N=1000 used on a continuing supply of lots of size 100 from the producer, that is, in a Type B sampling situation. Clearly rectification plans are meaningless on isolated lots, even though they might be 100 percent inspected if rejected, because there is no long-term average involved. The Type B probabilities of acceptance have already been calculated are listed in table which shows the calculation of the average total inspection (ATI) for m=1, 2, s=2,3,4,5,6,7,8 and 9. More tables can be calculated for different values of m and s. It is apparent that the ATI curve starts at 465.625 the sample size n=100, when μ =0.02 since no lots are 100 percent inspected and rises to 1000 when p=25 since all lots will be rejected and 100 percent inspected when the lots are completely defective. The ATI curve is shown below.

3. CONCLUSION

In this article, our work is mainly related to the Selection of Bayesian Multiple Deferred Sampling (BMDS-1) plan for quality level. Mathematical modeling of Average Outgoing Quality Level (AOQL) and Average Total Inspection (ATI) are completely derived and also analyzed. The tables are made elegant, handy and also made easy to use in industrial shop floor situations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Latha. Certain studies relating to Bayesian acceptance sampling plan. Ph.D Thesis, Department of Statistics, Bharathiar University, Tamilnadu, India; 2002.
2. Montgomery C Douglas. Statistical quality control: A modern introduction. 6th Edition, John Wiley and sons Inc., New York. 2009;644-646.
3. Wortham AW, Baker RC. Multiple deferred state sampling inspection. The International Journal of Production Research. 1976;14(6):719-731.
4. Latha M, Subbiah K. Bayesian Multiple Deferred State (BMDS-1) sampling plan with Weighted Poisson distribution. International Journal of Emerging Trends in Engineering and Development. 2014;3(4):275-282.
5. Soundararajan V, Vijayaraghavan R. Construction and selection of multiple dependent (deferred) state sampling plan. Journal of Applied Statistics. 1990;17(3): 397-409.
6. Subramani K, Govindaraju K. Selection of multiple deferred state MDS-1 sampling plan for given acceptable and limiting quality levels involving minimum risks. Journal of Applied Statistics. 1990;17(3): 431-434.
7. Varest R. A procedure of construct multiple deferred state sampling plans. Methods of Operation Research. 1982;37:477-485.
8. Suresh, et al. Construction and evaluation of performance measures for Bayesian chain sampling plan (BCH sp-1). Acta Cincm Ndica. 2007;33(4):16-35.
9. Dodge HF. Chain Sampling inspection plan. Industrial Quality Control. 1955;11: 10-13.
10. Rao CR. On discrete distributions arising out of methods of ascertainment. Sankhya Series, A. 1965;311-324.
11. Joan Del Castillo, Peres-Casany. Weighted Poisson distribution for under dispersion and over dispersion situation. Ann. Inst. Statist. Math. 1998;50(3):567-585.
12. Patil GP, Rao CR, Ratnaparki MV. On discrete weighted distributions and their use in model for observed data. Communication in Statistics Theory and Models. 1986;15(3):907-918.
13. Patil GP, Rao CR, Ratnaparki MV. Weighted distributions and size- biased sampling with applications to wildlife population and human families. Biometrics. 1978;34:179-189.
14. Irving W Burr. Engineering statistics and quality control. McGraw-Hill Book Company, Inc., New York. 1953;307-319.
15. Duncan AJ. Quality control and industrial statistics. 3rd Edition, Homewood, Illinois. 1970;200-201.
16. Hald A. Statistical theory of sampling inspection by attributes (Newyork, Academic Press); 1981.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/17133