Asian Research Journal of Mathematics

11(3): 1-15, 2018; Article no.ARJOM.45489

ISSN: 2456-477X

Solution of Partial and Integro-Differential Equations Using the Convolution of Ramadan Group Transform

Mohamed A. Ramadan[1*] and Asmaa K. Mesrega[1]

[1]*Department of Mathematics, Faculty of Science, Menoufia University, Shebin El-Kom, Egypt.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2018/45489
Editor(s):
(1) Dr. Hari Mohan Srivastava, Professor, Department of Mathematics and Statistics, University of Victoria, Canada.
Reviewers:
(1) Rahmatullah Ibrahim Nuruddeen, Federal University Dutse, Nigeria.
(2) Haci Mehmet Baskonus, Munzur University, Turkey.
Complete Peer review History: http://www.sciencedomain.org/review-history/27392

Received: 10 September 2018
Accepted: 18 November 2018
Published: 24 November 2018

Original Research Article

Abstract

Differential and integral as well as Partial integro-differential equations (PIDE) occur naturally in various fields of science, engineering and social sciences. In this article, the Ramadan group integral transform of the convolution is used to solve such types of equations. We propose a most general form of a linear PIDE with a convolution kernel. First, the PIDE is converted to an ordinary differential equation (ODE) using Ramadan group transform (RGT). Solving this ordinary differential equation and applying inverse RGT an exact solution of the problem is obtained. Illustrative examples are considered to demonstrate the applicability and the effectiveness of the proposed RG transform of convolution for solving integral and integro- differential equations. It is observed that the RGT is a simple, more general and reliable technique for solving such equations.

Keywords: Partial and integro-differential equations; the convolution of Ramadan Group transform; Ramadan Group transform.

2010 AMS: 34K05;34K28;65R10;56R20.

**Corresponding author: E-mail: ramadanmohamed13@yahoo.com, mramadan@eun.eg;*

1 Introduction

Real life phenomena are often modeled by ordinary and partial differential equations. Due to the local nature of ordinary differential operator (ODO), the models containing merely ODOs do not help in modeling memory and hereditary properties. One of the best remedies to overcome this drawback is the introduction of integral term in the model. The ordinary and partial differential equation along with the weighted integral of an unknown function gives rise to an integro-differential equation (IDE) or a partial integro-differential equation (PIDE) respectively. Analysis of such equations can be found in [1-4].

One of the most known methods to solve partial differential equations is the Ramadan Group transform (RGT) method [5-7] which is considered to be the generalisation of the known integral transforms as Laplace transform method [8,9] and Sumudu transform method [10]. This generalised integral transform is applied successfully for solving ordinary and partial differential equations easily, see for example Hadhoud, and Mesrega [11]. In fact, there are also some similar recent valuable new integral transforms, as ZZ integral transform and Abooha integral transform which are available in literature and applied for solving different forms of partial differential equations. The interested reader is referred, for example [12-15]. The sumudu integral transform is applied in variety of differential and integral equations as well as fractional ordinary differential equations, see for example [16]. The Natural transform decomposition method (NTDM) which has been constructed by combining Natural transform method (NTM) and Adomian decomposition method (ADM) has been proposed. This algorithm has been applied to obtain the approximate solutions for the modified Camassa-Holm equation (mCHE) and modified Degasperis-Procesi equation (mDPE), see [17].

2 Ramadan Group Transform and the Convolution Theorem

2.1 Ramadan Group Transformation (RGT) [5,6]

A new integral Ramadan Group transform (RGT) defined for functions of exponential order, was proclaimed. We consider functions in the set A, defined by:

$$A = \{ f(t) : \exists M, t_1, t_2 \succ 0 \, s.t. \left| f(t) \right| \prec Me^{\frac{|t|}{t_n}}, if\ t \in (-1)^n \times [0,\infty) \}$$

The RG transform is defined by

$$K(s,u) = RG[f(t);(s,u)] = \begin{cases} \int_0^\infty e^{-st} f(ut)dt & -t_1 < u \le 0 \,, \\ \int_0^\infty e^{-st} f(ut)dt & 0 \le u < t_2 \,. \end{cases}$$

This transform, which is a generalisation of Laplace and Sumudu transforms, is introduced by Ramadan et al. [6,18] and accidentally and unpredictably, it was also introduced by Khan and Khan under the name of N-Transform [19].

Consider $F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)dt$, $G(u) = \int_0^\infty e^{-t} f(ut)dt$ are the Laplace and Sumudu integral transforms respectively, then we can write the following theorems

Theorem 2.1.1 [6]

$K(s,1) = F(s)$, $K(1,u) = G(u)$ and $K(s,u) = \frac{1}{s} F(s)$.

Theorem 2.1.2 [6]

Suppose $K(s,u)$ is the Ramadan Group transform of the function $f(t)$ then we

can prove the following

$$RG(\frac{df(t)}{dt}) = \frac{sRG[f(t)] - f(0)}{u},$$

$$RG(\frac{d^2 f(t)}{dt^2}) = \frac{s^2 RG[f(t)] - sf(0) - uf_t(0)}{u^2},$$

and in general

$$RG(\frac{d^n f(t)}{dt^n}) = \frac{s^n RG[f(t)]}{u^n} - \sum_{k=0}^{n-1} \frac{s^{n-k-1} f^{(k)}(0)}{u^{n-k}}.$$

In fact, familiarity with the convolution operation is necessary for the understanding of many other topics that feature in this text such as the solution of partial differential equations (PDEs) and other topics that are outside it such as the use of Green's functions for forming the general solution of various types of boundary value problem (BVP).

In mathematics and especially, in functional analysis the convolution [20-22] is a mathematical operator of two functions f and g. In the next section we will present a brief profile of convolution.

Definition 2.1.3 Convolution of two functions

The convolution of piecewise continuous functions, $f(t), g(t) : R \to R$ is the function $f * g : R \to R$ and is defined by the integral

$$f * g = \int_0^t f(\tau) g(t - \tau) d\tau.$$

2.2 Algebraic properties of convolution

For every piecewise continuous functions, $f(t)$, $g(t)$ and $h(t)$ the following hold

1- Commutativity: $f * g = g * f$.
2- Associatively: $f * (g * h) = (f * g) * h$.
3- Distributive: $f * (g + h) = (f * g) + (f * h)$.
4- Associatively with scalar multiplication: $a(f * g) = (af) * g$.
5- Complex conjugation: $\overline{f * g} = \overline{f} * \overline{g}$.

The above properties of convolutions are easy to be verified, for example see the proof of first property of [11].

2.3 Convolution theorem of Ramadan Group Transform [21]

Theorem 2.3.1 Let $f(t)$ and $g(t)$ be two functions with Ramadan group transforms of $K_1(s,u)$ and $K_2(s,u)$, respectively. Then

$$RG[(f*g)(s,u)] = uK_1(s,u)K_2(s,u),$$

and

$$RG^{-1}[uK_1(s,u)K_2(s,u)] = f*g.$$

For proof of the theorem, see [21].

2.4 Application of RGT of the Convolution Theorem for Differential Equations

In this subsection some differential equations are solved to illustrate the use of convolution theorem with Ramadan group transform

Example 2.1

Consider the first order of differential equation

$$y' + 3y = \cos 3t \ , \tag{2.1}$$

with initial conditions

$$y(0) = 0 \, . \tag{2.2}$$

Taking Ramadan Group transform of equation (2.1) with respect to t

$$\frac{sRG[y] - y(0)}{u} + 3RG[y] = \frac{u}{s^2 + 9u^2} \ , \tag{2.3}$$

which reduced to

$$\frac{s}{u}RG[y] + 3RG[y] = \frac{u}{s^2 + 9u^2} \ , \tag{2.4}$$

which can be further written as

$$RG[y] = u\left(\frac{s}{s^2 + 9u^2}\right)\left(\frac{1}{s + 3u}\right) \ , \tag{2.5}$$

where equation (2.5) gives the form of the convolution theorem of Ramadan Group transform

Taking the inverse of RG transform of equation (2.5), we have

$$y(t) = e^{-3t} * \cos(3t) \, . \tag{2.6}$$

The convolution is

$$y(t) = e^{-3t} * \cos(3t) = \int_0^t e^{-3(t-\tau)} \cos(\tau)\, d\tau = e^{-3t} \int_0^t e^{3\tau} \cos(3\tau)\, d\tau \,. \tag{2.7}$$

Integrating by parts of equation (2.7), yield

$$\int_0^t e^{3\tau} \cos(3\tau)\, d\tau = \frac{1}{3} e^{3t} \cos(3t) - \frac{1}{3} + \int_0^t e^{3\tau} \sin(3\tau)\, d\tau \,, \tag{2.8}$$

$$\int_0^t e^{3\tau} \cos(3\tau)\, d\tau = \frac{1}{6} [e^{3t} \cos(3t) + e^{3t} \sin(3t) - 1] \;. \tag{2.9}$$

Substituting in equation (2.7) about the integral (2.9)

$$y(t) = e^{-3t} [\frac{1}{6} e^{3t} \cos(3t) + \frac{1}{6} e^{3t} \sin(3t) - \frac{1}{6}] \;. \tag{2.10}$$

Then

$$y(t) = \frac{1}{6} [\cos(3t) + \sin(3t)] - \frac{1}{6} e^{-3t} \,. \tag{2.11}$$

Example 2.2

Consider the second order differential equation

$$y''(t) + y(t) = t \;, \tag{2.12}$$

with initial condition

$$y(0) = 1 \,,\, y'(0) = 0 \;. \tag{2.13}$$

Taking Ramadan Group transform of equation (2.12) with respect to t

$$\frac{s^2 RG[y] - sy(0) - uy'(0)}{u^2} + RG[y] = \frac{u}{s^2} \;, \tag{2.14}$$

which reduced to

$$\frac{s^2}{u^2} RG[y] + RG[y] = \frac{u}{s^2} \;, \tag{2.15}$$

which can be further written as

$$RG[y] = \frac{s}{s^2 + u^2} + \frac{u^3}{s^2 (s^2 + u^2)} \;. \tag{2.16}$$

In the second term of the right hand side of equation (2.16), we note that, this term is the form of convolution theorem of Ramadan Group transform.

Setting $K_1(s,u)=\frac{u}{s^2}$ and $K_2(s,u)=\frac{u}{s^2+u^2}$ in equation (2.16) and taking inverse Ramadan Group transform, we have

$$y(t)=K^{-1}[\frac{s}{s^2+u^2}]+K^{-1}[uK_1(s,u)K_2(s,u)]\ , \tag{2.17}$$

$$y(t)=cos(t)+(t*sin(t)). \tag{2.18}$$

The convolution is

$$t*sin(t)=\int_0^t (t-\tau)sin(\tau)d\tau\ .. \tag{2.19}$$

Integrating by parts of equation (2.19), we have

$$\int_0^t (t-\tau)sin(\tau)d\tau=\left[-(t-\tau)cos(\tau)\right]_0^t-\int_0^t cos(\tau)d\tau=t-sin(t)\ .. \tag{2.20}$$

Substituting from equation (2.20) into equation (2.18), we find

$$y(t)=cos(t)-sin(t)+t\ . \tag{2.21}$$

Example 2.3

Consider the second order of differential equation

$$y''(t)+6y'+9y=sint \quad ,(t\geq 0) \tag{2.22}$$

with initial condition

$$y(0)=0\ ,\quad y'(0)=0\ . \tag{2.23}$$

Taking Ramadan Group transform of equation (2.22) with respect to t

$$\frac{s^2RG[y]-sy(0)-uy'(0)}{u^2}+6(\frac{sRG[y]-y(0)}{u})+9RG[y]=\frac{u}{s^2+u^2}\ , \tag{2.24}$$

which reduced to

$$\frac{s^2}{u^2}RG[y]+\frac{6s}{u}RG[y]+9RG[y]=\frac{u}{s^2+u^2}\ , \tag{2.25}$$

which can be further written as

$$RG[y] = \frac{u^3}{(s^2+u^2)(s+3u)^2} = u(\frac{u}{s^2+u^2})(\frac{u}{(s+3u)^2}). \quad (2.26)$$

Equation (2.26) gives the form of the convolution theorem of Ramadan Group transform

Taking the inverse of RG transform of equation (2.26), we have

$$y(t) = te^{-3t} * sin(t) = \int_0^t \tau e^{-3\tau} sin(t-\tau)d\tau. \quad (2.27)$$

This integral yield to integration by parts several times. The result follows from application of the formula:

$$\int_0^t \tau e^{-3\tau} sin(t-\tau) = \frac{-1}{10}\int_0^t e^{-3t} cos(t-\tau)d\tau + \frac{3}{10}\int_0^t e^{-3\tau} sin(t-\tau)d\tau + \frac{1}{10}te^{-3t}. \quad (2.28)$$

Thus the exact solution is

$$y(t) = \frac{e^{-3t}}{50}(5t+3) - \frac{3}{50}cos(t) + \frac{2}{25}sin(t). \quad (2.29)$$

3 Application of RGT of the Convolution Theorem for Differential Equations

In this section some integral equations are solved to illustrate the use of convolution theorem with Ramadan group transform

Example 3.1

Consider the following integral equation

$$y(t) = t + 2\int_0^t cos(t-\tau)y(\tau)d\tau. \quad (3.1)$$

We note that the second term of equation (3.1) gives the convolution of two functions $cos\,t$ and $y(t)$

$$y(t) = t + 2(cos(t) * y(t)). \quad (3.2)$$

Taking Ramadan Group transform of equation (3.2) with respect to t and using the convolution theorem, we have

$$RG[y(t);(s,u)] = K[y] = \frac{u}{s^2} + 2uK_1[cos(t)]K_2[y(t)]. \quad (3.3)$$

Assuming that $RG[y(t);(s,u)] = Y(s,u)$, so equation (3.3) is

$$Y(s,u) = \frac{u}{s^2} + 2u(\frac{s}{s^2+u^2})(Y(s,u)), \quad (3.4)$$

$$Y(s,u) = \frac{u}{(s-u)^2} + \frac{u^3}{s^2(s-u)^2}. \quad (3.5)$$

Equation (3.5) can be written in the form

$$Y(s,u)=\frac{u}{(s-u)^2}+[u(\frac{u}{s^2})(\frac{u}{(s-u)^2})] \quad . \tag{3.6}$$

Taking the inverse of RG transform of equation (3.6), we have

$$y(t)=te^t+(t*te^t). \tag{3.7}$$

The convolution of equation (3.7) is

$$t*te^t=\int_0^t(t-\tau)\tau e^\tau d\tau=t\int_0^t\tau e^\tau d\tau-\int_0^t\tau^2 e^\tau d\tau. \tag{3.8}$$

Integrating by parts of equation (3.7), we have

$$t*te^t=-te^t+t+2te^t-2e^t+2. \tag{3.9}$$

Substituting from equation (3.9) into equation (3.7), we have

$$y(t)=2te^t-2e^t+t+2=2e^t(t-1)+t+2. \tag{3.10}$$

Example 3.2

Consider the following integral equation

$$y(t)=1+\int_0^t y(\tau)sin(t-\tau)d\tau. \tag{3.11}$$

We note that the second term of equation (3.11) gives the convolution of two functions $y(t)$ and $sin(t)$

$$y(t)=1+[y(t)*sin(t)]. \tag{3.12}$$

Taking Ramadan Group transform of equation (3.12) with respect to t and using the convolution theorem, we have

$$RG[y]=\frac{1}{s}+u(RG[y(t)])(\frac{u}{s^2+u^2}), \tag{3.13}$$

$$RG[y]=\frac{s^2+u^2}{s^3}=\frac{1}{s}+\frac{u^2}{s^3}. \tag{3.14}$$

Taking the inverse of RG transform of equation (3.14), we have

$$y(t)=1+\frac{t^2}{2!}. \tag{3.15}$$

Example 3.3

Consider the following integral differential equation

$$y'(t) = t + \int_0^t y(t-\tau)\cos\tau \, d\tau \,, \tag{3.16}$$

with initial condition

$$y(0) = 0 \,. \tag{3.17}$$

We note that the second term of equation (3.16) gives the convolution of two functions $y(t)$ and $\cos t$

$$y'(t) = t + (y(t) * \cos(t)) \,. \tag{3.18}$$

Taking Ramadan Group transform of equation (3.18) with respect to t and using the convolution theorem, we have

$$\frac{sRG[y] - y(x,0)}{u} = \frac{u}{s^2} + u(RG[y])(\frac{s}{s^2+u^2}) \,, \tag{3.19}$$

which is reduced to

$$RG[y](\frac{s}{u} - \frac{us}{(s^2+u^2)}) = \frac{u}{s^2} \,. \tag{3.20}$$

which can be further written as

$$RG[y] = \frac{u^2}{s^3} + \frac{u^4}{s^5} \,. \tag{3.21}$$

Taking the inverse of RG transform of equation (3.21), we have

$$y(x,t) = \frac{t^2}{2!} + \frac{t^4}{4!} \,. \tag{3.22}$$

4 Application of RGT of the Convolution Theorem for Partial-Integro Differential Equations

In this section some partial -integro differential equations are solved to illustrate the use of Convolution theorem with Ramadan group transform

Example 4.1

Consider the following partial integro differential equation [23]

$$xy_x = y_{tt} + x\sin t + \int_0^t \sin(t-\tau)y(x,\tau)d\tau \,, \tag{4.1}$$

with initial condition

$$y(x,0)=0 \quad , \quad y_t(x,0)=x \; , \tag{4.2}$$

and boundary conditions

$$y(1,t)=t \; . \tag{4.3}$$

We note that the third term of the right hand side of equation (4.1) gives the convolution of two functions $\sin t$ and $y(x,t)$

$$xy_x = y_{tt} + x\sin t + (\sin t * y(x,t)) \; . \tag{4.4}$$

Taking Ramadan Group transform of equation (4.4) with respect to t and using the convolution theorem, we have

$$x\frac{d\bar{y}}{dx} = \frac{s^2\bar{y} - sy(x,0) - uy'(x,0)}{u^2} + x\left(\frac{u}{s^2+u^2}\right) + u\left(\frac{u}{s^2+u^2}\right)\bar{y} \; , \tag{4.5}$$

where $RG[\,y(x,t)] = \bar{y}$,

which the equation (4.5) can be further written as

$$\frac{d\bar{y}}{dx} - \frac{1}{x}\left(\frac{s^2}{u^2} + \frac{u^2}{s^2+u^2}\right)\bar{y} = \frac{-s^2}{u(s^2+u^2)} \; . \tag{4.6}$$

The equation (4.6) be linear differential equation in the form

$$\frac{d\bar{y}}{dx} + P(x)\bar{y} = Q(x) \; .$$

To solve this equation (4.6) first, we find

$$\mu(\bar{y}) = e^{\int P(x)dx} \; ,$$

$$\mu(\bar{y}) = e^{\int -\frac{1}{x}\left(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2}\right)dx} = e^{-\left(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2}\right)\int\frac{dx}{x}} = e^{-\left(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2}\right)\ln x} \; , \tag{4.7}$$

$$\mu(\bar{y}) = x^{-\left(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2}\right)} \; . \tag{4.8}$$

Second,

$$\frac{d}{dx}[\,\bar{y}(x,s).\mu(\bar{y})] = \mu(\bar{y}).Q(x),$$

$$\bar{y}(x,s)\,x^{-(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2})} = \frac{-s^2}{u(s^2+u^2)}\int x^{-(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2})}\,dx, \tag{4.9}$$

$$\bar{y}(x,s)\,x^{-(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2})} = \frac{-s^2}{u(s^2+u^2)(\frac{-s^2}{u^2}-\frac{u^2}{s^2+u^2}+1)}\,x^{-(\frac{s^2}{u^2}+\frac{u^2}{s^2+u^2})+1} + c\,. \tag{4.10}$$

Then the solution of equation (4.6) is

$$\bar{y}(x,s) = \frac{u}{s^2}x + c, \tag{4.11}$$

where c is a constant to be determined from (4.3).

Since $\bar{y}(x,t) = RG[\,y(x,t)]$,then

$$RG[\,y(1,t)] = RG[t] = \frac{u}{s^2} = \frac{u}{s^2} + c\ ,\quad c = 0, \tag{4.12}$$

from (4.12) into (4.11), we get

$$\bar{y}(x,s) = \frac{u}{s^2}x. \tag{4.13}$$

Taking the inverse of RG transform of equation (4.13), we get the exact solution as

$$y(x,t) = xt. \tag{4.14}$$

Example 4.2

Consider the following partial integro differential equation [23]

$$y_{tt}(x,t) = y_x(x,t) + 2\int_0^t (t-\tau)y(x,\tau)d\tau - 2e^x, \tag{4.15}$$

with initial condition

$$y(x,0) = e^x,\quad y_t(x,0) = 0, \tag{4.16}$$

and boundary condition

$$y(0,t) = \cos t. \tag{4.17}$$

We note that the second term of the right hand side of equation (4.15) gives the convolution of two functions t and $y(x,t)$

$$y_{tt} = y_x + 2(t * y(x,t)) - 2e^x. \tag{4.18}$$

Taking Ramadan group transform of equation (4.18) with respect to t and using the convolution theorem, we have

$$\frac{s^2\bar{y}-sy(x,0)-uy_t(x,0)}{u^2}=\frac{d\bar{y}}{dx}+2[u(\frac{u}{s^2})(\bar{y}(x,t))]-\frac{2}{s}e^x, \tag{4.19}$$

which can be written in the form

$$\frac{d\bar{y}}{dx}+(\frac{2u^2}{s^2}-\frac{s^2}{u^2})\bar{y}=(\frac{2}{s}-\frac{s}{u^2})e^x, \tag{4.20}$$

or

$$\frac{d\bar{y}}{dx}+(\frac{2u^2}{s^2}-\frac{s^2}{u^2})\bar{y}=(\frac{2u^2-s^2}{su^2})e^x. \tag{4.21}$$

To solve the equation (4.21) first, we find

$$\mu(\bar{y})=e^{\int(\frac{2u^2}{s^2}-\frac{s^2}{u^2})dx}=e^{(\frac{2u^2}{s^2}-\frac{s^2}{u^2})x}, \tag{4.22}$$

Second,

$$\frac{d}{dx}[\bar{y}(x,s).e^{(\frac{2u^4-s^4}{u^2s^2})x}]=(\frac{2u^2-s^2}{su^2})e^{(\frac{2u^4-s^4}{u^2s^2})x}e^x. \tag{4.23}$$

By integrating of equation (4.23), we get

$$\bar{y}(x,s).e^{(\frac{2u^4-s^4}{u^2s^2})}=(\frac{2u^2-s^2}{su^2})\int e^{(\frac{2u^4-s^4}{u^2s^2}+1)x}dx, \tag{4.24}$$

$$\bar{y}(x,s)=\frac{1}{e^{(\frac{2u^4-s^4}{u^2s^2})}}[(\frac{2u^2-s^2}{su^2})(\frac{1}{(\frac{2u^4-s^4}{u^2s^2}+1)})e^{(\frac{2u^4-s^4}{u^2s^2}+1)x}+c, \tag{4.25}$$

$$\bar{y}(x,s)=\frac{s(2u^2-s^2)}{(2u^2-s^2)(u^2+s^2)}e^x+c\,e^{-(\frac{2u^4-s^4}{u^2s^2})x}. \tag{4.26}$$

Then the solution of equation (4.21) is

$$\bar{y}(x,t)=\frac{s}{(s^2+u^2)}e^x+c\,e^{(\frac{s^4-2u^4}{s^2u^2})x}, \tag{4.27}$$

where c is a constant to be determined from the boundary condition (4.17).

$$RG[\cos t] = \frac{s}{s^2+u^2} + c \quad .$$

Then $c = 0$ and equation (4.27) is

$$\bar{y}(x,t) = \frac{s}{(s^2+u^2)} e^x . \tag{4.28}$$

Taking the inverse of RG transform of equation (4.28), we get the exact solution as

$$y(x,t) = e^x \cos t. \tag{4.29}$$

Example 4.3

Consider the following partial integro differential equation [23]

$$y_t - y_{xx} + y + \int_0^t e^{t-\tau} y(x,\tau) d\tau = (x^2+1)e^t - 2, \tag{4.30}$$

with initial conditions

$$y(x,0) = x^2 \; , \; y_t(x,0) = 1 \; , \tag{4.31}$$

and boundary conditions

$$y(0,t) = t \quad , \; y_x(0,t) = 0. \tag{4.32}$$

We note that the integral term of equation (4.30) gives the convolution of two functions e^t and $y(x,t)$

$$y_t - y_{xx} + y + (e^t * y(x,t)) = (x^2+1)e^t - 2. \tag{4.33}$$

Taking Ramadan Group transform of equation (4.33) with respect to t and using the convolution theorem, we have

$$\frac{s\bar{y} - y(x,0)}{u} - \frac{d^2\bar{y}}{dx^2} + \bar{y} + u(\frac{1}{s-u})\bar{y} = \frac{x^2+1}{s-u} - \frac{2}{s}. \tag{4.34}$$

Equation (4.34) can be written in the form

$$\frac{d^2\bar{y}}{dx^2} - \frac{s^2}{u(s-u)}\bar{y} = -x^2(\frac{s}{u(s-u)}) - \frac{1}{s-u} + \frac{2}{s}. \tag{4.35}$$

Similar to the case of examples 4.1 and 4.2, one can prove that the solution of equation (4.35) is $y(x,t) = x^2 + t$.

This is an exact solution of (4.30).

5 Conclusions

In this paper, Ramadan group integral transform of the convolution theorem is successfully applied to solve general linear differential, integral and partial integro –differential equations. The exact solutions are obtained after a few steps of calculations. The proposed method is a generalisation of any integral transform of convolution theorem available methods for this type of considered. The strong point of this article is in the high applicability to engineering problems.

Acknowledgement

The authors would like to thank the referees for valuable suggestions and comments, which helped the authors to improve this article substantially.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Appell JM, Kalitvin AS, Zabrejko PP. Partial integral operators and integro-differential equation, M. Dekker, New York; 2000.

[2] Bahuguna D, Dabas J. Existence and uniqueness of a solution to a PIDE by the method of lines. Electronic Journal of Qualitative Theory of Differential Equations. 2008;4:1-12.

[3] Pachapatte BG. On some new integral and integro –differential inequalities in two independent variables and their applications. Journal of Differential Equations. 1979;33:249-272.

[4] Yanik EG, Fairweather. Finite element methods for parabolic and hyperbolic partial integro-differential equations. Nonlinear Analysis: Theory, Method and Applications. 1988;12:785-809.

[5] Ramadan MA, Hadhoud AR. Ramadan Group (RG) transform coupled with projected differential transform for solving nonlinear partial differential equations. American Journal of Mathematical and Computer Modeling. 2017;2(2):39-47.

[6] Raslan KR, Ramadan MA, Talaat TS, Hadhoud AR. On a new general integral transform: Some properties and remarks. Journal of Mathematical and Computational Science. 2016;6(1):103-109.

[7] Adel R. Hadhoud, Asmaa K. Mesrega. Combination of Ramadan group and reduced differential transforms for partial differential equations with variable coefficients. Asian Research Journal of Mathematics. 2017;6(1):1-15.

[8] Eltayeb H, Kilicman A. A note on double Laplace transform and telegraphic equations. Abstract and Applied Analysis. 2013, Article ID 932578, 6 page http://dx.doi.org/10.1155/2013/932578.

[9] Dhunde RR, Waghmare GL. Double Laplace and its applications. International Journal of Engineering Research and Technology. 2013;2(12):1455-1462.

[10] Kilicman A, Eltayeb H. Some remarks on the Sumudu and Laplace transforms and Applications to differential equations. ISRN Applied Mathematics; 2012. DOI:10.5402/2012/591517.

[11] Hadhoud AR, Mesrega AK. Combination of Ramadan group and reduced differential transforms for partial differential equations with variable coefficients. Asian Research Journal of Mathematics. 2017;6(1):1-15.

[12] Aboodh KS. The new integral transform Aboodh transform. Global Journal of Pure and Applied Mathematics. 2013;9(1):35-43.

[13] Zain Ul Abadin Zafar, ZZ Transform, Method. International Journal of Advanced Engineering and Global Technology. 2016;4(1).

[14] Nuruddeen RI, Nass AM. Exact solutions of wave-type equations by the Aboodh decomposition method. Stochastic Modelling and Applications. 2017;21(1):23-30.

[15] Nuruddeen RI, Muhammad L, Nass AM, Sulaiman TA. A review of the integral transforms-based decomposition methods and their applications in solving nonlinear PDEs. Palestine Journal of Mathematics. 2018;1(7):262-280.

[16] Bulut H, Baskonus HM, Belgacem FBM. The analytical solutions of some fractional ordinary differential equations by Sumudu transform method. Abstract and Applied Analysis, Volume 2013, Article ID 203875, 6 pages, 2013.

[17] Baskonus HM, Bulut H. A comparison between NTDM and VIM for modified Camassa-Holm and modified Degasperis-Procesi equations. Nonlinear Studies the International Journal. 2015;22(4):601-611.

[18] Ramadan MA, Raslan KR, Hadhoud AR, Mesrega AK. A substitution method for partial differential equations using Ramadan group integral transform. Asian Research Journal of Mathematics. 2017;7(4):1-10.

[19] Khan ZH, Khan WA. N-transform – Properties and application. NUST Journal of Engineering Sciences. 2008;1:127-133.

[20] Jyoti T, Sachin B. Solving Partial Integro- Differential equations using Laplace transform method. American Journal of Computational and Applied Mathematics. 2012;2(3):101-104.

[21] Ramadan MA. The convolution for Ramadan group integral transform: Theory and applications. Journal of Advanced Trends in Basic and Applied Science. 2017;1(2):191-197.

[22] Hohlfeld RG, King JIF, Drueding TW, Sandri GVH. Solution of Convolution integral equations by the method of differential inversion. SIAM J. Applied Mathematics. 1993;53(1):154-167.

[23] Thorwe J, Bhalekar S. Solving partial Integro-Differential equations using Laplace transform method. American Journal of Computational and Applied Mathematics. 2012;2(3):101-1-4.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/27392