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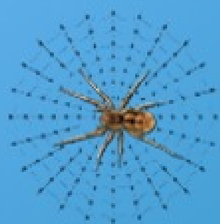

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A SEMI-ANALYTICAL SOLUTION FOR THE MOTION OF A LOW ALTITUDE EARTH SATELLITE UNDER J2-GRAVITY AND AIR DRAG PERTURBATIONS

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ABSTRACT

The motion of the low altitude Earth satellites is important for space applications, since these altitudes are crowded by a large number of artificial satellites. At this region the Earth's oblateness and the drag force play an important role and capture the dynamics of the problem. The present work investigates the motion of a low altitude Earth satellite under the combined effect of the Earth's gravity, up to the fourth zonal harmonic, and the drag force. The equations of motion are formulated under the considered force model. The problem under concern is treated using two different techniques, Cowell's and Average methods. We used the TLE data of the International Space Station (ISS) to compare the analytical method (average method) and the numerical Cowell's method. To better understanding the problem, we carried out several numerical explorations. A Mathematica code is constructed to simulate the numerical examples. Comparing the two methods, we found that Cowell's method gave more acceptable results.

Keywords: Perturbation Effects; Atmospheric drag; Oblate Earth; Cowell's Method; Average Method; Low Earth orbit (LEO).

1. Introduction

A perturbation is a deflection in the orbital motion of two body, one revolving around the other. The disparity of the masses of the bodies that revolve around each other leads to a difference between the actual path and the theoretical path of two body. There are some forces that Kepler did not take into account, such as drag and the non-spherical of the Earth. We cannot consider that the perturbations are minor, because there may affect greater than the attractive forces between the two body. The orbit of the satellite can be predicted by using some perturbation method. The average value of the orbital elements can be computed by many different ways. Analytical theory treated by many authors [1-11]. Spaceflight is given great credit for studying the effect of atmospheric drag on a satellite. The first to help and work in that is King-Hele [12]. He ignored other perturbations and one studied the effect drag atmospheric drag. Battin [13] reviews the change of the average values to the semi-major axis and eccentricity. Vallado [14] and Roy [15] deduced the change equations for both semi-major axis and eccentricity to obtain the secular-periodic terms rates of change of the orbital elements. Mittleman [16] solved the non-integral drag problem using the approximate mean. Vinh [17] used numerical methods to calculate the change of orbital elements in the equations of orbital motions. Most of the previous studies use the analytical solution for oblateness, drag, and vice versa to find the integration problem. The current work aims to construct the perturbed forces of Earth's zonal potential field and drag in the equations of motion and treat them analytically and numerically utilize average method and Cowell's method.

2. Dynamical system

2.1. Preliminaries

The equation of motion of two-body unperturbed problem is given by

$$\ddot{r} = -\frac{\mu}{r^3} r \tag{1}$$

where μ is gravitational parameter.

$$\ddot{r} = -\frac{\mu}{r^3} r + P \tag{2}$$

P represents additional forces as zonal harmonics and atmospheric drag

The Lagrange planetary equations (LPE) [13] are

$$\frac{da}{dt}=\frac{2}{na}\left(\frac{\partial\mathcal{R}}{\partial M}\right) \tag{3.1}$$

$$\frac{de}{dt}=\frac{1}{na^2}\frac{1-e^2}{e}\left(-\frac{1}{\sqrt{1-e^2}}\frac{\partial\mathcal{R}}{\partial\omega}+\frac{\partial\mathcal{R}}{\partial M}\right) \tag{3.2}$$

$$\frac{di}{dt}=\frac{1}{na^2\sqrt{1-e^2}\sin i}\left(-\frac{\partial\mathcal{R}}{\partial\Omega}+\cos i\frac{\partial\mathcal{R}}{\partial\omega}\right) \tag{3.3}$$

$$\frac{d\Omega}{dt}=\frac{1}{na^2\sqrt{1-e^2}\sin i}\left(\frac{\partial\mathcal{R}}{\partial i}\right) \tag{3.4}$$

$$\frac{d\omega}{dt}=\frac{1}{na^2\sqrt{1-e^2}}\left(\frac{1-e^2}{e}\frac{\partial\mathcal{R}}{\partial e}-\cot i\frac{\partial\mathcal{R}}{\partial i}\right) \tag{3.5}$$

$$\frac{dM}{dt}=\frac{1}{na^2}\left(-2a\frac{\partial\mathcal{R}}{\partial a}-\frac{1-e^2}{e}\frac{\partial\mathcal{R}}{\partial e}\right) \tag{3.6}$$

where $n=\sqrt{\frac{\mu}{a^3}}$ is the mean motion and $\mathcal{R}$ is perturbing potential.

The Gaussian planetary equations are used in most the perturbed force (conservative and non-conservative) acting on the orbit. The Gaussian planetary equations are represented in **RSW** frame, i.e., $\nabla F=\acute{R}\,\hat{u}_r+S\,\hat{u}_\theta+W\hat{u}_w$. Then resulting equations are [13]:

$$\dot{a}=\frac{2}{n\sqrt{1-e^2}}\left(e\sin f\acute{R}+\frac{p}{r}\acute{S}\right) \tag{4.1}$$

$$\dot{e}=\frac{\sqrt{1-e^2}}{na}\left[\sin f\acute{R}+\left(\cos f+\frac{e+\cos f}{1+e\cos f}\right)\acute{S}\right] \tag{4.2}$$

$$\frac{di}{dt}=\frac{r\cos(\omega+f)}{na^2\sqrt{1-e^2}}W \tag{4.3}$$

$$\dot{\Omega}=\frac{r\sin(\omega+f)}{na^2\sqrt{1-e^2}\sin i}\acute{W} \tag{4.4}$$

$$\dot{\omega}=\frac{\sqrt{1-e^2}}{nae}\left[-\cos f\acute{R}+\sin f\left(1+\frac{r}{p}\right)\acute{S}\right]-\frac{r\cot i\sin(\omega+f)}{\sqrt{\mu p}}\acute{W} \tag{4.5}$$

$$\dot{M}=\frac{1}{na^2e}\left[(p\cos f-2er)\acute{R}-(p+r)\sin f\acute{S}\right] \tag{4.6}$$

where $p = a(1 - e^2)$, f is the true anomaly, and **R, S, W**. are perturbing accelerations.

2.2. Semi-analytical Solution

In this section, the zonal potential and atmospheric drag will be solved by using the semi analytical solution.

The Zonal Part: The zonal harmonics part is given by [13]

$$\mathcal{R}_{zonal}(r,\psi) = \frac{\mu}{r}\left[1 - \sum_{b=2}^{\infty}\left(\frac{r_e}{r}\right)^b J_b P_b(\sin\psi)\right] \tag{5}$$

$\mathcal{R}$ is the perturbing potential . Where J_b , $b = 2, 3, \ldots$ are the gravitational coefficients, ψ is the latitude, $\sin\psi = \sin i \sin(\omega + f)$, $u = \omega + f$ is the argument of latitude, r_e is equatorial radius of Earth and $P_b(\psi)$ Legendre polynomial, can be expressed as

$$P_b(\psi) = \frac{1}{2^b b!}\frac{d^b}{d\psi^b}\left(\psi^2 - 1\right)^b$$

The average of R can be calculated as

$$\langle\mathcal{R}_{zonal}\rangle = \frac{1}{T}\int_0^T \mathcal{R}\,dt = \frac{1}{2\pi}\int_0^{2\pi}\mathcal{R}\,dM \tag{6}$$

where the relation between dM and df is

$$dM = \frac{r^2}{a^2\sqrt{1-e^2}}df$$

Substitute this equation into equation (6) the average $\mathcal{R}_{zonal}$ becomes,

$$\langle\mathcal{R}_{zonal}\rangle = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\sqrt{1-e^2}}\left(\frac{r}{a}\right)^2 df = \mathcal{R}_{sec} + \mathcal{R}_{long} \tag{7}$$

(a) Second Degree Expansion: From equation (5), the Earth's potential function which includes the perturbation due to the oblateness of the Earth can be defined as,

$$\mathcal{R} = -\frac{\mu r_e^2}{r^3}J_2\frac{3\sin^2\psi - 1}{2} \tag{8}$$

The perturbation due to the oblateness of the Earth causes a precession of the line of nodes around the normal to the ecliptic plane. If the orbital perturbations are analyzed for one orbit, the average of a function over one period can be used as

$$\langle\mathcal{R}_2\rangle = -\frac{1}{4}\frac{\mu r_e^2}{a^3(1-e^2)^{\frac{3}{2}}}J_2\left(3\sin^2 i - 2\right) \tag{9}$$

Substituting equation (9) into equation (3), the mean orbital elements affected by the J_2 perturbation are equal to,

$$\dot{a} = \dot{e} = \frac{di}{dt} = 0 \tag{10.1}$$

$$\dot{\Omega} = -\frac{3}{2\left(1-e^2\right)^2} nJ_2 \left(\frac{r_e}{a}\right)^2 \cos i \qquad (10.2)$$

$$\dot{\omega} = \frac{3}{4\left(1-e^2\right)^2} nJ_2 \left(\frac{r_e}{a}\right)^2 \left(5\cos^2 i - 1\right) \qquad (10.3)$$

$$\dot{M} = \frac{3}{4\left(1-e^2\right)^{\frac{3}{2}}} nJ_2 \left(\frac{r_e}{a}\right)^2 \left(3\cos^2 i - 1\right) \qquad (10.4)$$

$\dot{a}, \dot{e}, \dot{i}, \dot{\Omega}, \dot{\omega}$ and $\dot{M}$ are the mean orbital elements for the semi major axis, eccentricity, inclination angle, RAAN, argument of perigee, and mean anomaly. The semi major axis, the eccentricity, and the inclination angle are not affected by secular perturbation of J_2. The only orbital elements affected are the RAAN, the argument of perigee, and the mean anomaly which cause a rotation and precession of the orbit. The rotation and precession can be compared to the spinning of a symmetrical top with an inclined rim. The nodal precession is observed in the rate of change of the RAAN. If the inclination angle is equal to 90° degrees, there is no precession or regression of the orbit. For $0 < I < 90°$, the node precesses toward the west it called a regression node. If $90° < I < 180°$ the node precesses toward the east it called a prograde node. The argument of perigee has a rate of precession about the polar axis which causes a motion of the major axis of the ellipse. There are two inclination angles for which there is no precession (or regression) of the argument of perigee, where i = 63.4 and 116.6 degrees. If $i < 63.4°$, the line of apsides precesses in the direction of the orbit motion. For $63.4° < I < 116.6°$, the line of apsides precesses in opposite sense to the orbital motion.

(b) Third Degree Expansion: We start our calculation of $\mathcal{R}$ by expanding up to third degree, We the Legendre polynomials P_3 (sin ψ). $\langle\mathcal{R}_3\rangle$ becomes

$$\langle\mathcal{R}_3\rangle = \frac{3}{8}\frac{\mu r_e{}^3 e}{a^4\left(1-e^2\right)^{\frac{5}{2}}} J_3 \left(-5\sin^2 i + 4\right)\sin i \sin\omega \qquad (11)$$

The final result is:

$$\dot{a} = 0 \qquad (12.1)$$

$$\dot{e} = -\frac{3}{16} nJ_3 \left(\frac{r_e}{a}\right)^3 \left[\frac{3+5\cos 2i}{\left(1-e^2\right)^2}\right] \cos\omega \sin i \qquad (12.2)$$

$$\frac{di}{dt} = -\frac{3}{16} ne J_3 \left(\frac{r_e}{a}\right)^3 \left[\frac{3+5\cos 2i}{\left(1-e^2\right)^3}\right] \cos i \cos\omega \qquad (12.3)$$

$$\dot{\Omega} = -\frac{3}{15} ne J_3 \left(\frac{r_e}{a}\right)^3 \left[\frac{\cos i + 15\cos 3i}{\left(1-e^2\right)^3}\right] \csc i \sin\omega \qquad (12.4)$$

$$\dot{\omega} = \frac{3}{64} n J_3 \left(\frac{r_e}{a}\right)^3 \sin\omega \csc i \left[\frac{-1-3e^2-4\cos 2i+5\left(1+7e^2\right)\cos 4i}{e\left(1-e^2\right)^3}\right] \qquad (12.5)$$

$$\dot{M} = \frac{3}{32} n J_3 \left(\frac{r_e}{a}\right)^3 \sin\omega \left[\frac{\left(1+12e^2\right)\left(\sin i + 5\sin 3i\right)}{e\left(1-e^2\right)^{\frac{5}{2}}}\right] \qquad (12.6)$$

(c) Fourth Degree Expansion: For degree 4, we use the Legendre polynomial $P_4(\sin\psi)$ and proceed as before. This leads to

$\langle \mathcal{R}_4 \rangle = -\frac{1}{1024a^5(1-e^2)^{7/2}} 3r_e{}^4 \mu(18 + 27e^2 + 40\text{Cos}[2i] + 60e^2\text{Cos}[2i] + 70\text{Cos}[4i] + 105e^2\text{Cos}[4i] + 30e^2\text{Cos}[2\omega] - 35e^2\text{Cos}[2(-2i+\omega)] + 20e^2\text{Cos}[2(-i+\omega)] + 20e^2\text{Cos}[2(i+\omega)] - 35e^2\text{Cos}[2(2i+\omega)])J_4$

(13)

Similarly, $\langle \mathcal{R}_2 \rangle, \langle \mathcal{R}_3 \rangle$ and the final result is:

$$\dot{a} = 0 \qquad (14.1)$$

$$\dot{e} = \frac{15}{64} e\, n\, J_4 \left(\frac{r_e}{a}\right)^4 \sin^2 i \sin 2\omega \left[\frac{5 + 7\cos 2i}{\left(1 - e^2\right)^3}\right] \qquad (14.2)$$

$$\frac{di}{dt} = \frac{15}{256} e^2\, n\, J_4 \left(\frac{r_e}{a}\right)^4 \sin 2\omega \left[\frac{10\sin 2i + 7\sin 4i}{(1-e^2)^4}\right] \qquad (14.3)$$

$$\dot{\Omega} = \frac{15}{128} n\, J_4 \left(\frac{r_e}{a}\right)^4 \left[\frac{(2+3e^2)(9\cos i + 7\cos 3i) - 2e^2\cos 2\omega\,(5\cos i + 7\cos[3i))}{(1-e^2)^4}\right] \qquad (14.4)$$

$$\dot{\omega} = \frac{15}{1024} n\, J_4 \left(\frac{r_e}{a}\right)^4 \left[\frac{-27(4+5e^2) + 2(-6+5e^2)\cos 2\omega + 4\cos 2i\left(-52 - 63e^2 + 2(-2+7e^2)\cos 2\omega\right)}{\left(1-e^2\right)^4}\right] \qquad (14.5)$$

$$\dot{M} = \frac{15}{1024} n\, J_4 \left(\frac{r_e}{a}\right)^4 \left[\frac{(8+9e^2)(9+20\cos 2i + 35\cos 4i) + 8(2+15e^2)(5+7\cos 2i)\cos 2\omega \sin^2 i}{(1-e^2)^{\frac{7}{2}}}\right] \qquad (14.6)$$

Finally, the total effect of the gravitational field of the earth is,

$$\dot{a}_z = 0 \qquad (15.1)$$

$$\dot{e}_z = \frac{3\, r_e{}^3 \mu\, \text{Cos}[\omega]\text{Sin}[i]\left(-2a(-1+e^2)(3+5\text{Cos}[2i])J_3 + 5e\, r_e(5+7\text{Cos}[2i])\, J_4\text{Sin}[i]\text{Sin}[\omega]\right)}{32a^7\left(-1+e^2\right)^3 n} \qquad (15.2)$$

$$\frac{di}{dt_z} = \frac{3\, e\, r_e{}^3 \mu\, \text{Cos}[i]\text{Cos}[\omega]\left(-2a(-1+e^2)(3+5\text{Cos}[2i])J_3 + 5\, e\, r_e(5+7\,\text{Cos}[2i])J_4\,\text{Sin}[i]\text{Sin}[\omega]\right)}{32a^7\left(-1+e^2\right)^4 n} \qquad (15.3)$$

$\dot{\Omega}_z = (3\, r_e{}^2\, \mu\, (-5\, e^2\, r_e{}^2(5\,\text{Cos}[i] + 7\,\text{Cos}[3i])\text{Cos}[2\omega]J_4 + \text{Cos}[i](-32a^2(-1+e^2)^2 J_2 + 5(2+3e^2)\, r_e{}^2(1+7\text{Cos}[2i])\, J_4) - 2ae(-1+e^2)r_e(\text{Cos}[i] + 15\text{Cos}[3i])\text{Csc}[i]J[3]\text{Sin}[\omega]))/ 64a^7(-1+e^2)^4 n$

(15.4)

$\dot{\omega}_z = \frac{1}{1024a^7 e(-1+e^2)^4 n} 3r_e{}^2\mu(128a^2e(-1+e^2)^2(3+5\text{Cos}[2i])J_2 - 5er_e{}^2(27(4+5e^2) + 4(52 + 63e^2)\text{Cos}[2i] + 7(28+27e^2)\text{Cos}[4i])J_4 + 10er_e{}^2(-6+5e^2+4(-2+7e^2)\text{Cos}[2i] + 7(2 + 9e^2)\text{Cos}[4i])\text{Cos}[2\omega]J_4 + 16a(-1+e^2)r_e(-1-3e^2-4\text{Cos}[2i] + 5(1 + 7e^2)\text{Cos}[4i])\text{Csc}[i]J_3\text{Sin}[\omega])$

(15.5)

$\dot{M}_z = (-3r_e{}^2\mu(-384a^2e(-1+e^2)^2(1+3\text{Cos}[2i])J_2 + 5e(8+9e^2)r_e{}^2(9+20\text{Cos}[2i] + 35\text{Cos}[4i])J_4 + 40e(2+15e^2)r_e{}^2(5+7\text{Cos}[2i])\text{Cos}[2\omega]J_4\text{Sin}[i]^2 + 32a(-1+e)(1+e)(1 + 12e^2)r_e\, J_3(\text{Sin}[i] + 5\text{Sin}[3i])\text{Sin}[\omega]))/1024a^7e(1-e^2)^{7/2}n$

(15.6)

Atmospheric Drag

The development of the drag coefficient model for various solar activities is given by Cook [18]. Marcos has studied the precision of the drag model for LEO satellites orbit the density [19]. Moe et al studied the geometric shapes of the drag coefficient by using orbital measurements [20]. Storz et al. worked to develop a high-resolution model for drag of low-orbit satellites [21]. Most of the effects of the drag coefficient on LEO satellites have been reviewed using most theories by Prieto [22]. The force acting on the drag model can be represented as

$$F_{drag} = -\frac{1}{2}\left(\frac{S\,C_D}{m}\right)\rho\,(V - v)\|V - v\| \tag{16}$$

where *m* is the satellite mass, *S* is the effective cross-sectional area, C_D is the drag coefficient, The coefficient $\beta = \left(\frac{S\,C_D}{m}\right)$ known as the ballistic coefficient. The vector V is the velocity of the satellite. The vector v is the atmospheric velocity. If the atmosphere was with the beginning of the revolutions of the Earth and it would be spherical, then $v = [0\ 0\ \omega_e]^T r$, where ω_e is the Earth's rotation rate and **r** is the position vector. Using the Herring approach, the atmospheric density may be written as [13]

$$\rho = \rho_0 \exp\left(\frac{r_{p0} - r}{H}\right) \tag{17}$$

Where ρ_0 is the initial value of the atmospheric density, r_{p0} is the initial perigee radius and *H* is the density scale height. The drag force vector F_{drag}, written in the **NTW** frame [9].

$$\acute{N} = 0 \tag{18.1}$$

$$\acute{T} = -\frac{1}{2}K_1\,n^2 a^2 \rho\,\frac{1 + 2e\cos f + e^2}{1 - e^2} \tag{18.2}$$

$$\acute{W} = -\frac{1}{2}K_2\,n\,a\,\rho\,r\cos(\omega + f)\sin i\left(\frac{1 + 2e\cos f + e^2}{1 - e^2}\right)^{\frac{1}{2}} \tag{18.3}$$

Where $K_1 = \left(\frac{S_1\,C_D}{m}\right)Q$, $K_2 = \left(\frac{S_2\,C_D}{m}\right)\omega_e\sqrt{Q}$, and $Q = \left(1 - \frac{r_{p0}\,\omega_e\cos i}{v_{p0}}\right)$.

The variables S_1 and S_2 are the cross-sectional areas, and v_{p0} is the initial velocity at perigee. Using the transformation from NTW to RSW, the drag components can be obtained [14].

$$\acute{R} = \frac{e\sin f}{\left(1 + e^2 + 2e\cos(f)\right)^{\frac{1}{2}}}\acute{T} + \frac{1 + e\cos f}{\left(1 + e^2 + 2e\cos(f)\right)^{\frac{1}{2}}}\acute{N} \tag{19.1}$$

$$\acute{S} = \frac{1 + e\cos f}{\left(1 + e^2 + 2e\cos(f)\right)^{\frac{1}{2}}}\acute{T} - \frac{e\sin f}{\left(1 + e^2 + 2e\cos(f)\right)^{\frac{1}{2}}}\acute{N} \tag{19.2}$$

Substituting equations (18) and (19) into equation (4) give us:

$$\dot{a}_D = -\frac{K_1\,n\,a^2}{(1 - e^2)^{\frac{3}{2}}}\rho\left(1 + 2e\cos f + e^2\right)^{\frac{3}{2}} \tag{20.1}$$

$$\dot{e}_D = -\frac{K_1\, n\, a}{(1-e^2)^{\frac{1}{2}}}\, \rho\, (\cos f + e)\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}} \tag{20.2}$$

$$\frac{di}{dt_D} = -\frac{K_2\, a}{4\,(1 + e\cos f)^2}\, \rho \sin i\,[1 + \cos(2\omega + 2f)](1 - e^2)\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}} \tag{20.3}$$

$$\dot{\Omega}_D = -\frac{K_2\, a}{4\,(1 + e\cos f)^2}\, \rho \sin(2\omega + 2f)\,(1 - e^2)\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}} \tag{20.4}$$

$$\dot{\omega}_D = -\frac{K_1\, n\, a}{e\,(1-e^2)^{\frac{1}{2}}}\, \rho \sin f\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}} - \cos i\; \dot{\Omega}_{drag} \tag{20.5}$$

$$\dot{M}_D = -\frac{K_1\, n\, a}{e\,(1 + e\cos f)}\, \rho \sin f\left(1 + e\cos f + e^2\right)\left(1 + 2e\cos f + e^2\right)^{\frac{1}{2}} \tag{20.6}$$

Equations (20) can be solved by modified Bessel functions depend on the atmospheric density [23, 24], as a result the secular and the long-periodic terms is

$$\dot{a}_D = -K_1\, \rho_0\, n\, a^2\left[1 + e^2\left(\frac{3}{4} + \frac{a}{H} + \frac{a^2}{4H^2}\right) + O(e^3)\right] \exp\left(\frac{r_{p0} - a}{H}\right) \tag{21.1}$$

$$\dot{e}_D = -K_1\, \rho_0\, n\, a\left[\frac{e}{2} + \frac{a\,e}{2H} + O(e^3)\right] \exp\left(\frac{r_{p0} - a}{H}\right) \tag{21.2}$$

$$\frac{di}{dt_D} = -\frac{1}{4}K_2\, \rho_0\, a \sin i\left[1 + e^2\left(\frac{3}{4} - \frac{a}{H} + \frac{a^2}{4H^2}\right) + \left(\frac{11}{8} - \frac{a}{H} + \frac{a^2}{8H^2}\right) e^2 \cos 2\omega + O(e^3)\right] \exp\left(\frac{r_{p0} - a}{H}\right) \tag{21.3}$$

$$\dot{\Omega}_D = -\frac{1}{4}K_2\, \rho_0\, a\left[\left(\frac{11}{8} - \frac{a}{H} + \frac{a^2}{8H^2}\right) e^2 \sin 2\omega + O(e^3)\right] \exp\left(\frac{r_{p0} - a}{H}\right) \tag{21.4}$$

$$\dot{\omega}_D = -\cos i \,.\, \dot{\Omega} \tag{21.5}$$

$$\dot{M}_D = -\frac{3}{4}K_1\, \rho_0\, n^2\, a\left[1 + e^2\left(\frac{3}{4} + \frac{a}{H} + \frac{a^2}{4H^2}\right) + O(e^3)\right] \exp\left(\frac{r_{p0} - a}{H}\right) \times (t - t_0) \tag{21.6}$$

3. Numerical Solutions

3.1. Cowell's method

P.H. Cowell [25] discovered this simple and direct method of using all perturbation in the early twentieth century. This method is used to write all the effects of the perturbation on the motion system with respect to the two-body problem, and we can solve that numerically, the equation Become [25].

$$\ddot{r} = -\frac{\mu}{r^3} r + P \tag{22}$$

where P is perturbed forces

We can change the second order differential equation into a first order differential equation by numerical methods (Runge-kutta) [26]. We get the following equations

$$\mathrm{r} = -\frac{\mu}{\mathrm{r}^3} r + \mathrm{P} \Rightarrow \begin{cases} \ddot{\mathrm{x}} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{x} = \begin{cases} \dot{\mathrm{x}} = \mathrm{v_x} \\ \dot{\mathrm{v}}_\mathrm{x} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{x} + \mathrm{P} \end{cases} \\ \ddot{\mathrm{y}} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{y} = \begin{cases} \dot{\mathrm{y}} = \mathrm{v_y} \\ \dot{\mathrm{v}}_\mathrm{y} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{y} + \mathrm{P} \end{cases} \\ \ddot{\mathrm{z}} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{z} = \begin{cases} \dot{\mathrm{y}} = \mathrm{v_y} \\ \dot{\mathrm{v}}_\mathrm{z} = -\dfrac{\mu}{\mathrm{r}^3}\mathrm{z} + \mathrm{P} \end{cases} \end{cases}$$

$$= \begin{cases} \dot{x} = v_x = f_1(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ \dot{y} = v_y = f_2(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ \dot{z} = v_z = f_3(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ \dot{v}_x = -\frac{\mu}{r^3} x = f_4(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ \dot{v}_y = -\frac{\mu}{r^3} y = f_5(x, y, z, \dot{x}, \dot{y}, \dot{z}) \\ \dot{v}_z = -\frac{\mu}{r^3} z = f_6(x, y, z, \dot{x}, \dot{y}, \dot{z}) \end{cases} \quad (23)$$

where $\mathrm{r} = (\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2)^{\frac{1}{2}}$.

Equations (23) can be solved numerically using Runge-Kutta method which in the form of first order ordinary differential equation (ODE). The ODE solved, the initial position and initial velocity must be specified initial conditions. The state vector position and velocities of a satellite can be converted into orbital elements [27].

4. Results

The Russian Zarya satellite (ISS ZARYA) was launched in 1998. Its altitude is about 411 Km. (http://celestrak.com) The Orbital elements data can be extracted using two line element, initial time use the data:

```
ISS (ZARYA)
1 25544U 98067A   19351.54008334  .00016717  00000-0  10270-3 0  9070
2 25544  51.6378 172.3255 0007343  42.7724 3176.3997 15.50134307 3696
```

The orbital elements data can be extracted using two line element, final time use the data:

```
ISS (ZARYA)
1 25544U 98067A   19361.08141748  .00016717  00000-0  10270-3 0  9044
2 25544  51.6419 125.0498 0006268  72.9805 287.2031  15.49786732 5178
```

These two methods (LPE Method, Cowell's Method) represent two orbit classes for which the computation of oblate and drag is.

Table 1: The initial values of LEO satellite (ISS ZARYA)

a	6794.14 km	x	−6730.76239814029 km
e	0.0007343	y	898.4415337922493 km
i	51.6378°	z	10.696630262069181 km
Ω	172.3255°	$\dot{x}$	−0.616144045818917 km/sec
ω	42.7724°	$\dot{y}$	−4.716250831074117
M	317.3997°	$\dot{z}$	6.009083139725322 km/sec

Table 2: Physical parameters of the earth [14, 25].

J_2	$1082.62668355 \times 10^{-6}$
J_3	$-2.53265648533 \times 10^{-6}$
J_4	$-1.61962159137 \times 10^{-6}$
r_e	$6378.137\ km$
μ	$398600\ {km^3}/{sec^2}$
ρ_o	$3.725 \times 10^{-12}\ {kg}/{m^3}$
H	$58.515\ km$
β	0.044

Table 3: The orbital elements of ISS in LPE and Cowell Methods

	I-TLE	II **-LPE**	III **-Cowell's**	$\Delta(I - II)$	$\Delta(I - III)$
a	6795.14	6794.1336760	6794.1330107	1.0063239	1.0069892
e	0.0006268	0.0011225	0.0007311	−0.0004957	−0.0001043
i	51.6419	51.6377998	51.6378026	0.0041001	0.0040973
Ω	125.0498	120.3575371	172.3254941	4.6922628	−47.2756941
ω	72.9805	54.6627293	42.2604173	18.3177706	30.7200826
M	287.2031	316.9371120	326.6647785	−29.7340120	−9.7276664

Table (3) Shows solutions the orbital elements by using the LPE Method and Cowell's Method. We calculate the different between them during ten days. The column (**I**) is the finial TLE, the column (**II**) is the result of the (LPE) and the column (**III**) is the result of the (Cowell's Method). The column $\Delta(I - II)$ is the difference between TLE and result of the (LPE). The column $\Delta(I - III)$ is the difference between finial TLE and result of the (Cowell's Method). The sub figures (1) represent the propagation of orbital elements for ISS in ten days.

5. Conclusions

In this paper the effects of zonal harmonics (J_2 Up to J_4) and atmospheric drag force on low satellite orbits are studied. The equation of two body problem under pervious perturbing forces is formulated. The equations of motion are solved by two methods which are the semi-analytical solution (Average Method) and Numerical solution (Cowell's Method). A program code MATHEMATICA Language is constructed to treat the solutions (ISS). The outcome of a numerical orbit integration of Cowell's method and average Method are compared with the orbital elements of TLE, which we found the results from Cowell's Method more acceptable.

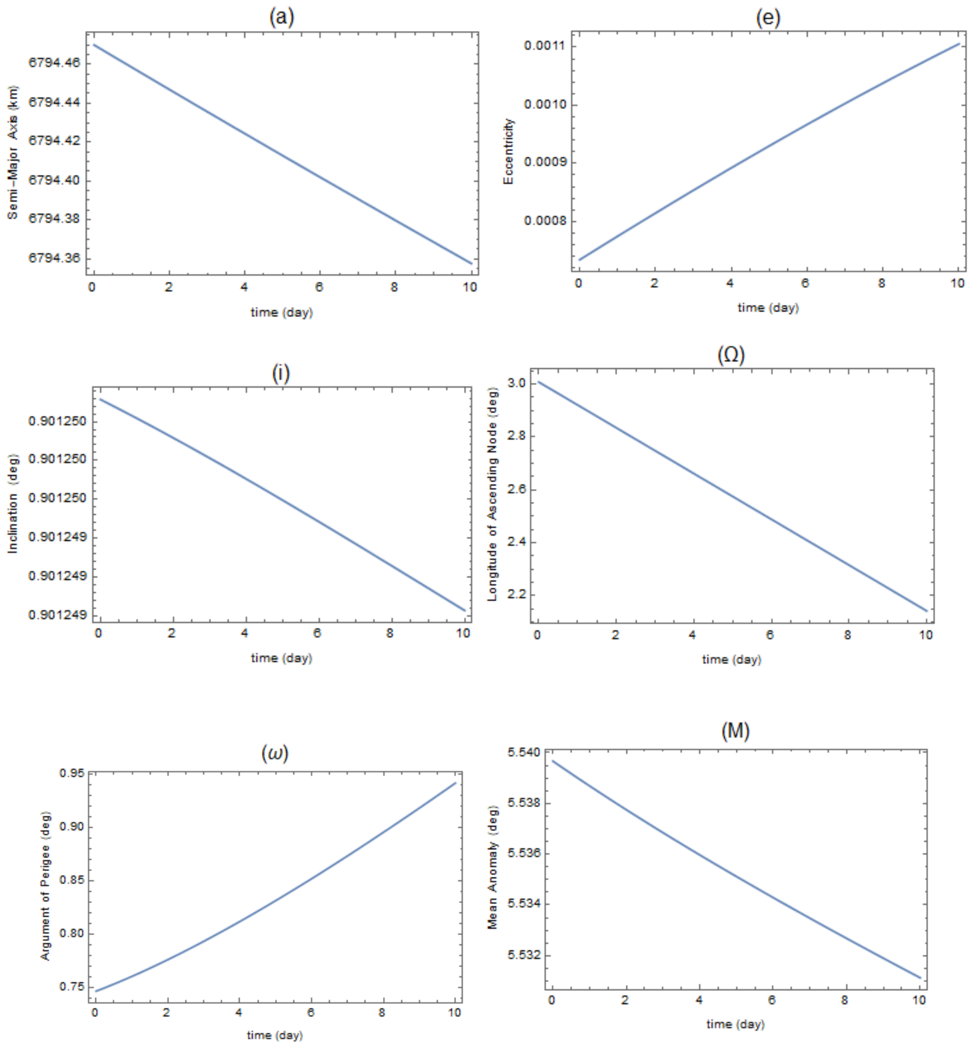

Fig. 1. The variation of orbital elements for ISS

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الحل النصف تحليلى للأقمار الصناعية المنخفضة تحت تاثير المجال التجاذبى للارض وقوة مقاومة الهواء

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الملخص:

دراسة حركة الاقمار الصناعية فى الارتفاعات المنخفضة هام جدا فى التطبيقات الفضائية. يعتبر انبعاج الارض وتاثير الغلاف الجوى ذا اهمية كبيرة فى هذه الارتفاعات المنخفضة.فى الدراسة الحالية حققنا حركة الاقمار الصناعية فى الارتفاعات المنخفضة تحت تاثر قوى انبعاج الارض حتى الرتبة الرابعة وكذلك تاثير مقاومة الهواء. معادلات الحركة كونت باستخدام نموذج القوى. طبقنا فى هذه الدراسة الحل النصف تحليلى باستخدام معادلات لاجرانج الكوكبية والحل العددى باستخدام طريقة كاوول. استخدمنا بيانات القمر الصناعى للمحطة الفضائية الدولية وقارنا هذه البيانات بالنتائج النموذجين المستخدمين وجدنا ان نتائج الطريقة العددية اقرب للبيانات المرصودة من المحطة الدولية.