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A Binary Block Code Generated by JU-Algebras

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Studies on the binary block codes generated by some ordered algebraic structures have been the interest of many researchers. In this paper, we study the binary block code generated by an arbitrary JU-algebra and investigate some of its properties. For this intent, we introduce the notion of a JU-function ϕ on a nonempty set P into a JU-algebra X, and by using that concept, j-functions and j-subsets of P for an arbitrary element j on a JU-algebra X are investigated. Furthermore, we define a new order on the generated code C based on the JU-algebra X, and show that every finite JU-algebra with its order and the corresponding generated code C with the defined order have the same structures. Finally, we generate a JU-algebra from a particular set of binary block code.

Keywords: JU-algebra; JU-function; j-function; j-subset; binary block code; coding theory.

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1 Introduction

The transmutation of information was one of the engineering issues of the twentieth century. In 1948 and 1950, Shannon [1] and Hamming [2] provided a framework for resolving the issue. As a result, coding theory was established and information could be transmitted over the noisy channel. The idea of coding theory is to provide a method for converting information into bits of zeros and ones such that errors in the received data are minimized.

An error-correcting code that stores data in blocks is known as block code in coding theory. In 2011, Jun and Song [3] applied coding theory in ordered algebraic structures and discussed its applications. During the last few years, researchers have been interested on binary block codes generated by various types of ordered algebraic structures.

In 2009, Leerawat and Prabpayak [4] presented the notion of KU-algebra. After a few years, Ali et al. [5] and Ansari et al. [6] introduced and examined the idea of JU-algebra as a generalization of KU-algebras. However, the concept of this class of algebra was previously presented by Leerawat and Prabpayak [7] under the name "pseudo KU-algebra or PKU. Since then, studies on generalization of KU-algebra have been the interest of numerous researchers.

In this study, the method used in generating binary block code in BL-algebras, KU-algebras, and BCK-algebras will be applied to JU-algebras. Relationships between the generated code and JU-algebras are obtained analogous to the results found in [3], [8]-[11].

2 Preliminary Notes

Some definitions of the concepts covered in this study are included below. You may refer on the remaining terms and definitions in [6].

Definition 2.1. [6] **(JU-algebra)** A nonempty set X with binary operation $*$ and a constant 1 is said to be a JU-algebra if it satisfies the following for any $x, y, z \in X$,

(JU_1) $(y * z) * [(z * x) * (y * x)] = 1$,

(JU_2) $1 * x = x$,

(JU_3) $x * y = y * x$ implies $x = y$.

For brevity, a JU-algebra $(X, *, 1)$ shall be denoted by X.

Example 2.1. Let $X_1 = \{1, 2, 3, 4, 5\}$ be a set with binary operation $*$ defined by the following Cayley table:

$*$	1	2	3	4	5
1	1	2	3	4	5
2	1	1	3	4	5
3	1	2	1	4	5
4	1	1	3	1	5
5	1	1	1	1	1

Then X_1 is a JU-algebra.

Lemma 2.2. [6] For a given JU-algebra X, define a relation "$\ll$ "on X by $y \ll x$ if and only if $x * y = 1$ where $x, y \in X$. Then $(X, \ll)$ is a partially ordered set.

Lemma 2.3. [6] If X is a JU-algebra, then the following hold for any $x, y, z \in X$:

(J_4) $x \ll y$ implies $y * z \ll x * z$,

(J_5) $x \ll y$ implies $z * x \ll z * y$,

(J_6) $(z * x) * (y * x) \ll y * z$,

(J_7) $(y * x) * x \ll y$,

(J_8) $x * x = 1$,

(J_9) $z * (y * x) = y * (z * x)$,

(J_{10}) If $(x * y) * y = 1$, then X is a KU-algebra,

(J_{11}) $(y * x) * 1 = (y * 1) * (x * 1)$.

Given the codeword c, the *Hamming weight* $w(c)$ of a codeword c is the number of nonzero components in the codeword. The *Hamming distance* between two codewords c_1 and c_2, denoted by $d(c_1, c_2)$ is the number of places in which the codewords c_1 and c_2 differ. In other words, $d(c_1, c_2)$ is the Hamming weight of the vector $c_1 - c_2$, representing the component-wise difference of the vectors c_1 and c_2. The *minimum Hamming distance* of a code C is the minimum distance between any two codewords in the code C, that is, $d(C) = \min\{d(x, y)|x \neq y, x, y \in C\}$.

A code with code length n, a total of M codewords, and minimum distance d shall be denoted by (n, M, d).

3 Main Results

In this section, the notions of JU-functions on a nonempty set P based on a JU-algebra X, the j-functions, and the j-subsets of P for an arbitrary element $j \in X$, will be introduced. Moreover, the structures of the graphs of JU-algebra X with respect to its order $\ll$ and the JU-code C based on X with its order $\preceq$ will be presented.

3.1 JU-Functions on a Nonempty Set P

Definition 3.1. **(JU-function on P)** Let P be a nonempty set and X be a JU-algebra. Any function $\phi : P \to X$ is called a JU-function on P.

Definition 3.2. **(j-subset of P)** Let P be a nonempty set and X be a JU-algebra. For a JU-function $\phi : P \to X$ on P and each $j \in X$, $P_j \coloneqq \{p \in P | j * \phi(p) = 1\}$. Here, P_j is called a j-subset of P.

Definition 3.3. **(j-function of ϕ)** Let P be a nonempty set and X be a JU-algebra. For a JU-function $\phi : P \to X$ on P and each $j \in X$, define $\phi_j : P \to \{0, 1\}$ for each $p \in P$ as follows:

$$\phi_j(p) = \begin{cases} 1, & \text{if } j * \phi(p) = 1, \\ 0, & \text{otherwise.} \end{cases}$$

The function ϕ_j is called a j-function of ϕ.

Example 3.1. *By using Example 2.1, for a set $P = \{p_1, p_2, p_3, p_4, p_5\}$, the function $\phi : P \to X_1$ defined by*

$$\phi(p_1) = 1, \phi(p_2) = 2, \phi(p_3) = 3, \phi(p_4) = 4, \text{ and } \phi(p_5) = 5$$

is a JU-function on P, and the j-subsets of P for each $j \in X_1$ are as follows:

$$P_1 = \{p_1\}, P_2 = \{p_1, p_2\}, P_3 = \{p_1, p_3\}, P_4 = \{p_1, p_2, p_4\}, \text{ and } P_5 = P.$$

In addition, for each $j \in X_1$, the j-functions of ϕ are shown in the following table:

ϕ_j	p_1	p_2	p_3	p_4	p_5
ϕ_1	1	0	0	0	0
ϕ_2	1	1	0	0	0
ϕ_3	1	0	1	0	0
ϕ_4	1	1	0	1	0
ϕ_5	1	1	1	1	1

Remark 3.1. Let P be a nonempty set and X be a JU-algebra. For a JU-function $\phi : P \to X$ on P, $p \in P_{\phi(p)}$ for all $p \in P$.

The following proposition shows the relationship between JU-function on P and its j-functions and j-subsets of P for each $j \in X$.

Proposition 3.1. *Let $\phi : P \to X$ be a JU-function on a nonempty set P based on X, where X is a JU-algebra. Then the function ϕ can be described by its j-functions and j-subsets of P for each $j \in X$ and for all $p \in P$, as the infimum of the following sets:*

$$\phi(p) = inf\{j \in X | p \in P_j\}.$$

In other words, $\phi(p) = inf\{j \in X | \phi_j(p) = 1\}$.

Proof. Let $p \in P$ such that $\phi(p) = j$. By Remark 3.1, $p \in P_j$. Thus, $j \in S = \{j_1 \in X | p \in P_{j_1}\}$. If $j' \in S$, then $p \in P_{j'}$ which means that $j \ll j'$. Hence, $\phi(p) = \inf S$. □

Proposition 3.2. *Let X be a JU-algebra and $\phi : P \to X$ be a JU-function on a nonempty set P based on X. If $j_1 \ll j_2$ for all $j_1, j_2 \in X$, then $P_{j_1} \subseteq P_{j_2}$.*

Proof. Let $j_1, j_2 \in X$ such that $j_1 \ll j_2$. By Lemma 2.2, $j_2 * j_1 = 1$. If $u \in P_{j_1}$, then $j_1 * \phi(u) = 1$ which means that $\phi(u) \ll j_1$. By (J_5), $j_2 * \phi(u) \ll j_2 * j_1$ and by (JU_2),

$$1 = (j_2 * j_1) * (j_2 * \phi(u)) = 1 * (j_2 * \phi(u)) = j_2 * \phi(u).$$

Thus, $u \in P_{j_2}$ and so, $P_{j_1} \subseteq P_{j_2}$. □

Theorem 3.2. *Let $\phi : P \to X$ be a JU-function on P. Then the following holds:*

(*i*) *for all $p_1, p_2 \in P, \phi(p_1) \neq \phi(p_2) \iff P_{\phi(p_1)} \neq P_{\phi(p_2)}$; and*

(*ii*) *for all $j \in X$ and $p \in P, p \in P_j \iff P_{\phi(p)} \subseteq P_j$.*

Proof. Let $\phi : P \to X$ be a JU-function on P.

(*i*) Let $p_1, p_2 \in P$ such that $\phi(p_1) \neq \phi(p_2)$. Then by (JU_3), $\phi(p_1) * \phi(p_2) \neq 1$ or $\phi(p_2) * \phi(p_1) \neq 1$. Now, $P_{\phi(p_1)} = \{u \in P | \phi(p_1) * \phi(u) = 1\}$ and $P_{\phi(p_2)} = \{u \in P | \phi(p_2) * \phi(u) = 1\}$. Thus, $p_2 \in P_{\phi(p_2)} \setminus P_{\phi(p_1)}$ or $p_1 \in P_{\phi(p_1)} \setminus P_{\phi(p_2)}$ and so, $P_{\phi(p_1)} \neq P_{\phi(p_2)}$.

Conversely, let $p_1, p_2 \in P$ such that $P_{\phi(p_1)} \neq P_{\phi(p_2)}$. Suppose on the contrary that $\phi(p_1) = \phi(p_2)$. If $u \in P_{\phi(p_1)}$, then $\phi(p_1) * \phi(u) = 1$. Since $\phi(p_1) = \phi(p_2)$, it follows that $1 = \phi(p_1) * \phi(u) = \phi(p_2) * \phi(u)$ which means that $u \in P_{\phi(p_2)}$. Hence, $P_{\phi(p_1)} \subseteq P_{\phi(p_2)}$. Similarly, if $v \in P_{\phi(p_2)}$, then $v \in P_{\phi(p_1)}$. Thus, $P_{\phi(p_1)} = P_{\phi(p_2)}$ which is a contradiction. Therefore, $\phi(p_1) \neq \phi(p_2)$.

(*ii*) Let $j \in X$ and $p \in P$ such that $p \in P_j$. Then $j * \phi(p) = 1$ which means that $\phi(p) \ll j$. By Proposition 3.2, $P_{\phi(p)} \subseteq P_j$.

Conversely, let $j \in X$ and $p \in P$ such that $P_{\phi(p)} \subseteq P_j$. Since $\phi(p) * \phi(p) = 1$, it follows by (J_8) that $p \in P_{\phi(p)}$. Thus, $p \in P_j$ since $P_{\phi(p)} \subseteq P_j$ by assumption.

□

Theorem 3.2 part (*ii*) shows that the converse of Proposition 3.2 is true. Thus, we have the following corollary.

Corollary 3.3. *Let $\phi : P \to X$ be a JU-function on a nonempty set P based on X, where X is a JU-algebra. Then for all $p_1, p_2 \in P$, $\phi(p_2) * \phi(p_1) = 1 \iff P_{\phi(p_1)} \subseteq P_{\phi(p_2)}$.*

For a JU-algebra X and JU-function $\phi : P \to X$ on a nonempty set P, define the set P_X as follows:

$$P_X := \{P_j | j \in X\}.$$

Proposition 3.3. *Let $\phi : P \to X$ be a JU-function on a nonempty set P. Then*

$$P = \cup\{P_j | j \in X\}.$$

Proof. Clearly, $\cup\{P_j | j \in X\} \subseteq P$. Now, let $p \in P$ and $j \in X$ such that $\phi(p) = j$. Then by using (J_8), $1 = j * j = j * \phi(p)$ which means that $p \in P_j$. That is, there exists $j \in X$ such that $p \in P_j \subseteq \cup\{P_j | j \in X\}$. Therefore, $P = \cup\{P_j | j \in X\}$. □

Let $\phi : P \to X$ be a JU-function on a nonempty set P. Define a relation $\approx$ on X by

$$j_1 \approx j_2 \iff P_{j_1} = P_{j_2} \tag{3.1}$$

for all $j_1, j_2 \in X$. Then the relation $\approx$ is an equivalence relation on X.

For an arbitrary element $j \in X$, define the sets $\phi(P)$ and $\{j\}_\ll$ as follows:

$$\phi(P) := \{j \in X | \phi(p) = j \text{ for some } p \in P\},$$
$$\{j\}_\ll := \{u \in X | u \ll j\} = \{u \in X | j * u = 1\}.$$

The relationships between an equivalence relation $\approx$ and the sets $\phi(P)$ and $\{j\}_\ll$ are described in the following theorem.

Theorem 3.4. *For a JU-function $\phi : P \to X$ on a nonempty set P and the elements $j_1, j_2 \in X$, we have the following assertion:*

$$j_1 \approx j_2 \iff \phi(P) \cap \{j_1\}_\ll = \phi(P) \cap \{j_2\}_\ll$$

Proof. Let $j_1, j_2 \in X$.
Suppose $j_1 \approx j_2$. Then $P_{j_1} = P_{j_2}$. Let $u \in \phi(P) \cap \{j_1\}_\ll$. Since $u \in \phi(P)$, it means that $u = \phi(p)$ for some $p \in P$ and so, $\phi(p) \in \{j_1\}_\ll$. Hence, $j_1 * \phi(p) = 1$ which means that $p \in P_{j_1}$. Since $P_{j_1} = P_{j_2}$, $p \in P_{j_2}$ and so, $j_2 * \phi(p) = 1$. That is, $\phi(p) \in \{j_2\}_\ll$. Thus, $u = \phi(p) \in \phi(P) \cap \{j_2\}_\ll$. Similarly, if $v \in \phi(P) \cap \{j_2\}_\ll$, then $v \in \phi(P) \cap \{j_1\}_\ll$. Therefore, $\phi(P) \cap \{j_1\}_\ll = \phi(P) \cap \{j_2\}_\ll$.
Conversely, suppose $\phi(P) \cap \{j_1\}_\ll = \phi(P) \cap \{j_2\}_\ll$. Let $u \in P_{j_1}$. Then $j_1 * \phi(u) = 1$ which means that $\phi(u) \in \phi(P)$ and $\phi(u) \in \{j_1\}_\ll$. Hence, $\phi(u) \in \phi(P) \cap \{j_1\}_\ll$. Since $\phi(P) \cap \{j_1\}_\ll = \phi(P) \cap \{j_2\}_\ll$, it follows that $\phi(u) \in \phi(P) \cap \{j_2\}_\ll$. That is, $\phi(u) \in \phi(P)$ and $\phi(u) \in \{j_2\}_\ll$. Thus, $j_2 * \phi(u) = 1$ and so, $u \in P_{j_2}$. Similarly, if $v \in P_{j_2}$, then $v \in P_{j_1}$. Therefore, $P_{j_1} = P_{j_2}$ and so, $j_1 \approx j_2$. □

3.2 Code Generated by a JU-Algebra

Let $\approx$ be the equivalence relation on X defined in Equation (3.1) and let $[j]$ denotes an equivalence class containing j for any $j \in X$. Then $[j] := \{k \in X | j \approx k\}$.

Example 3.5. *By using the j-functions of ϕ on Example 3.1, the equivalence classes of X_1 are $[1] = \{1\}$, $[2] = \{2\}$, $[3] = \{3\}$, $[4] = \{4\}$, $[5] = \{5\}$. Hence, we have five different equivalence classes, which are $[1], [2], [3], [4], [5]$.*

Definition 3.4. For $n \in \mathbb{N}$, let $P = \{p_1, p_2, ..., p_n\}$ and X be a finite JU-algebra. Let $\phi : P \to X$ be a JU-function on a nonempty set P and let $j \in X$. The codeword generated by $[j]$ is $c_j = c_1c_2 \cdots c_n$ where $c_i = \phi_j(p_i)$ with $p_i \in P$. The set of all codewords generated by all the equivalence classes on X is the JU-code based on X and shall be denoted by C_X.

Example 3.6. *By using the j-functions of ϕ on Example 3.1, we obtain*

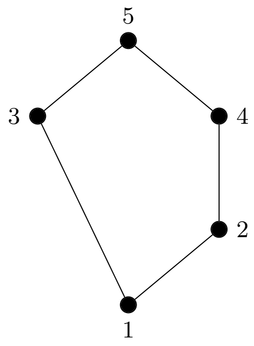
$$c_1 = 10000, c_2 = 11000, c_3 = 10100, c_4 = 11010, c_5 = 11111.$$

Hence, the total number of codewords is 5 ($M = 5$) and the binary block code of length $n = 5$ is $C_{X_1} = \{10000, 11000, 10100, 11010, 11111\}$. Moreover, the minimum Hamming distance of C_{X_1} is 1, that is, ($d(C_{X_1}) = 1$).

Definition 3.5. Let $c_j = j_1 j_2 \cdots j_n$ and $c_k = k_1 k_2 \cdots k_n$ be two codewords belonging to a binary block code C of length n. An order relationship $\preceq$ on a set of codewords belonging to a binary block code C of length n is as follows:

$$c_j \preceq c_k \iff j_i \leq k_i \text{ for } i = 1, 2, ..., n.$$

Example 3.7. *By using the j-functions of ϕ on Example 3.1, the generated binary block code C_{X_1} is $C_{X_1} = \{10000, 11000, 10100, 11010, 11111\}$. Hence, by using Lemma 2.2 for a JU-algebra X_1 and Definition 3.5 for a JU-code C_{X_1} based on X_1, we conclude that the graph of X_1 concerning the order $\ll$ and the code C_{X_1} with respect to the order $\preceq$ have the same structures. For instance, in Example 3.1, we have*



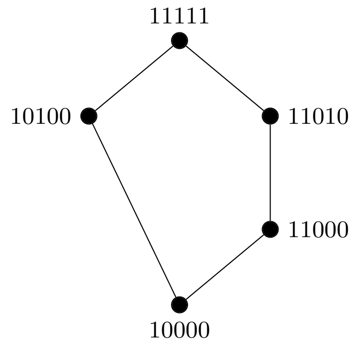


***Fig. 1. Graphs of** $(X_1, \ll)$ **and** $(C_{X_1}, \preceq)$*

Theorem 3.8. *Let $(X, *, 1)$ be a finite JU-algebra and $|X| = n$, where $n \in \mathbb{N}$. Then X determines a binary block code C of length n (namely JU-code) such that the graph of X with respect to its order $\ll$ and the graph of JU-code C with respect to the order $\preceq$ have the same structures.*

Proof. Let $X = \{j_1, j_2, ..., j_n\}$. Define $\phi : X \to X$ by $\phi(j_i) = j_i$ for all $i = 1, 2, ..., n$. Suppose $\frac{X}{\approx}$ be a set of all equivalence classes of the elements of X with respect to the equivalence relation $\approx$ defined in Equation (3.1). That is,

$$\frac{X}{\approx} = \{[j] | j \in X\}, \text{ where } [j] = \{j_1 \in X | P_j = P_{j_1}\}$$

Define the mapping $\psi : \frac{X}{\approx} \to C$ by $\psi([j_i]) = c_{j_i}$, whereby using Definition 3.4, we have
$c_{j_i} = \phi_{j_i}(j_1)\phi_{j_i}(j_2) \cdots \phi_{j_i}(j_n)$ for $i = 1, 2, ..., n$. Then ψ is a well-defined monomorphism.

Next, we show that ψ preserves order, suppose $j_i, j_k \in X$ such that $j_i \ll j_k$, for $1 \ll i, k \ll n$. Then by Proposition 3.2, we have $X_{j_i} \subseteq X_{j_k}$. If $j \in X$ and $j \in X_{j_i}$, then $\phi_{j_i}(j) = 1$. Since $X_{j_i} \subseteq X_{j_k}$, it follows that $j \in X_{j_k}$ and so, $\phi_{j_k}(j) = 1$. Hence, $\phi_{j_i}(j) \ll \phi_{j_k}(j)$. Thus, in this case, $c_{j_i} \preceq c_{j_k}$. If $j \in X$ and $j \notin X_{j_i}$, then

$\phi_{j_i}(j) = 0$. Since $\phi_{j_k}(j) = 1$ if $j \in X_{j_k}$ and $\phi_{j_k}(j) = 0$ if $j \notin X_{j_k}$, it follows that $\phi_{j_i}(j) \ll \phi_{j_k}(j)$. Hence, $c_{j_i} \preceq c_{j_k}$. Thus, if $j_i \ll j_k$, then $\psi(j_i) \preceq \psi(j_k)$. Therefore, $(X, \ll)$ and $(C, \preceq)$ have the same structures. □

Example 3.9. Let $X_2 = \{1, 2, 3, 4, 5, 6\}$ be a JU-algebra with binary operation $*$ given by the following Cayley table:

*	1	2	3	4	5	6
1	1	2	3	4	5	6
2	1	1	3	4	5	6
3	1	1	1	4	5	6
4	1	1	1	1	5	6
5	1	1	1	4	1	6
6	1	1	1	1	1	1

Define $\phi : X_2 \to X_2$ by

$$\phi(1) = 1,\ \phi(2) = 2,\ \phi(3) = 3,\ \phi(4) = 4,\ \phi(5) = 5, \text{ and } \phi(6) = 6.$$

Then

ϕ_j	1	2	3	4	5	6
ϕ_1	1	0	0	0	0	0
ϕ_2	1	1	0	0	0	0
ϕ_3	1	1	1	0	0	0
ϕ_4	1	1	1	1	0	0
ϕ_5	1	1	1	0	1	0
ϕ_6	1	1	1	1	1	1

Clearly, we have six different equivalence classes, which are [1], [2], [3], [4], [5], and [6]. Thus, using Definition 3.4, we have

$$c_1 = 100000,\ c_2 = 110000,\ c_3 = 111000,\ c_4 = 111100,\ c_5 = 111010,\ c_6 = 111111.$$

That is, the total number of codewords is 6 ($M = 6$) and the binary block code of length $n = 6$ is:

$$C_{X_2} = \{100000, 110000, 111000, 111100, 111010, 111111\}.$$

Moreover, the graph of C_{X_2} using the order $\preceq$ is the same with the graph of X_2 with the order $\ll$ as shown below:

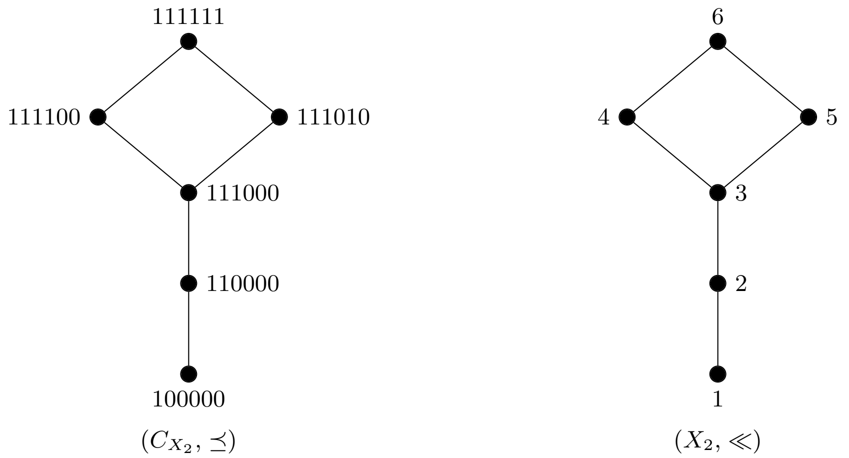


Fig. 2. Graphs of $(C_{X_2}, \preceq)$ and $(X_2, \ll)$

If $|C_X| \neq |X|$, then it is impossible for C_X and X to have the same structures. Now, we construct a JU-algebra from a particular set of binary block code.

Theorem 3.10. *Let $C = \{c_1, c_2, ..., c_n\}$ be a finite binary block code of length n such that $c_1 \preceq c_2 \preceq \cdots \preceq c_n$, where $n \in \mathbb{N}$. Then C determines a JU-algebra X such that $|X| = n$ and the graph of C with respect to the order $\preceq$ and the graph of X with respect to its order $\ll$ have the same structures.*

Proof. Let $X = \{1, 2, ...n\}$. Define $\phi : C \to X$ by $\phi(c_i) = i$. Then ϕ is an isomorphism. Next, we show that $X = \{1, 2, ..., n\}$ is a JU-algebra. Define an operation $*$ on X by

$$j * k = \begin{cases} 1, & \text{if } j \geq k, \\ k, & \text{if } j < k. \end{cases}$$

Then $(X, *, 1)$ is a JU-algebra.

Moreover, we show that ϕ preserves order. If $c_i \preceq c_j$, then $i \ll j$ for all $i, j \in \{1, 2, ..., n\}$. Observe that $c_i \preceq c_j$ implies $i \leq j$ which means $j * i = 1$ and so, $i \ll j$. Furthermore, the graphs of $(C, \preceq)$ and $(X, \ll)$ are shown below:

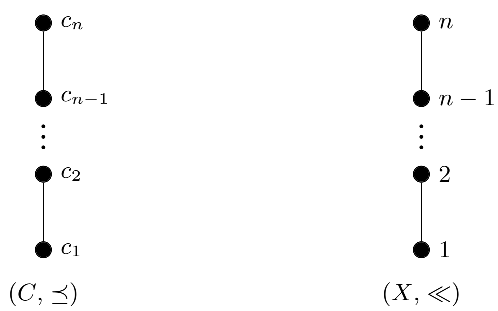


Fig. 3. Graphs of $(C, \preceq)$ and $(X, \ll)$

□

Example 3.11. *Let* $C_{X_3} = \{c_1 = 1000, c_2 = 1100, c_3 = 1110, c_4 = 1111\}$. *Take* $X_3 = \{1, 2, 3, 4\}$. *Using the binary operation* $*$ *given in the proof of Theorem 3.10, with Cayley table shown below:*

$*$	1	2	3	4
1	1	2	3	4
2	1	1	3	4
3	1	1	1	4
4	1	1	1	1

$(X_3, *, 1)$ *is a* JU*-algebra.*

4 Conclusion and Recommendation

In this article, the binary block code generated by JU-algebra were studied. Results parallel to [8], [3], [9] were generated. It is worth noting that the JU-code is not linear. For future research, it would be interesting to find a condition for JU-algebra so that its corresponding JU-code is linear. In general, the authors would like to investigate how the properties of JU-algebras affect its corresponding JU-code.

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Competing Interests

Authors declare that they have no competing interests.

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