British Journal of Mathematics & Computer Science
6(2): 79-90, 2015, Article no.BJMCS.2015.063
ISSN: 2231-0851

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Taylor Series Approach for Solving Multi-level Large Scale Fractional Programming Problem with Stochastic Parameters in Constraints

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Article Information
DOI: 10.9734/BJMCS/2015/13568
Editor(s):
(1) Qiankun Song, Department of Mathematics, Chongqing Jiaotong University, China.
Reviewers:
(1) Shu-Tao Ai, Department of Physics, School of Science, Linyi University, P.R. China.
(2) Asma Thamir Ibraheem, Al-Nahrain University, College of Engineering, Baghdad, Iraq.
Complete Peer review History:
http://www.sciencedomain.org/review-history.php?iid=732&id=6&aid=7387

Original Research Article

Received: 23 August 2014
Accepted: 01 October 2014
Published: 17 December 2014

Abstract

This paper presents a solution algorithm to solve a multi-level large scale fractional programming problem with individual chance constraints (CH-MLLSFP). We assume that there is randomness in the right-hand side of the constraints only and that the random variables are normally distributed. The basic idea in treating (CH-MLLSFP) is to convert the probabilistic nature of this problem into a deterministic multi-level large scale fractional programming problem (MLLSFPP). A solution of multi-level large scale fractional programming problem is presented using aTaylor series to avoid the complexity of fractional nature. An illustrative example is discussed to demonstrate the correctness of the proposed solution method.

Keywords: Large scale problems;stochastic programming; three-level programming.

2010 Mathematics Subject Classification: 90C06; 90C20; 90C99.

1 Introduction

Decision problems of chance- constrained or stochastic optimization arise when certain coefficients of an optimization model are not fixed or known but instead, to some extent, probabilistic quantities.

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In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extreme problems under incomplete information about the random parameters is called stochastic programming.The word ”stochastic” derives from the Greed ($\sigma\tau o\chi\eta\alpha\sigma\tau\iota\chi$ to aim, to guess) and means ”random” or ”chance.” The antonym is ”sure,” ”deterministic,” or ”certain.” A deterministic model predicts a single outcome from a given set of circumstances. ([1],[2],[3]).

Multilevel optimization problems have attracted considerable attention from the scientific and economic community in recent years. The multilevel system has extensive existences in management fields. Usually, this kind of problems can be solved by using different mathematical programming techniques ([4], [5]).

In large scale programming which closely describes and represents the real world decision situations, various factors of the real system should be reflected in the description of the objective function and constraints.Naturally these objective function and constraints involve many parameters and the experts may assign them different values ([6],[7]).After the publication of the Dantzig and Wolfe decomposition method([8]).

Fractional programming problem, which has been used as an important planning tool for the last four decades, is applied to different disciplines such as engineering, business, economics. . . etc. Fractional programming problem is generally used for modeling real life problems with objective such as profit/cost, inventory/sales, actual cost/standard cost . . . etc([9]).

This paper is organized as follows: we start in Section 2 by formulating the model of a multilevel large scale fractional programming problem with stochastic parameters in the constraints.In Section 3, deterministic multi-level large scale fractional programming problem.In Section 4, objective functions are transformed by using Taylor series. In Section 5,the decomposition method of large scale three level linear programming problem is presented. An algorithm for solving a multi-level large scale fractional programming problem (MLLSFPP) with stochastic parameters in constrain is suggested in Section 6. In Section 7, An algorithm flowchart for (CH-MLLSFPP). In addition, a numerical example is provided in Section 8 to clarify the results and the solution algorithm. Finally, Table of symbol, conclusion and future works are reported in Section 9.

2 Problem Formulation and Solution Concept

Multi-level large scale fractional programming problem (MLLSFPP)with random parameters in the right-hand side of the constrains may be formulated as follows:

[First Level]

$$\underset{x_1,x_2}{Max}\ F_1(x) = \underset{x_1,x_2}{Max}\ \sum_{j=1}^{m} \frac{b_{1j}^T x_j + \alpha_{1j}}{d_{1j}^T x + \beta_1}\ (j = 1, 2, \dots, N_1) \tag{1}$$

Where $x_3, \dots, x_m$ solves

[Second Level]

$$\underset{x_3,x_4}{Max}\ F_2(x) = \underset{x_3,x_4}{Max}\ \sum_{j=1}^{m} \frac{b_{2j}^T x_j + \alpha_{2j}}{d_{2j}^T x + \beta_2}\ (j = 1, 2, \dots, N_2) \tag{2}$$

Where $x_5, \dots, x_m$ solves

[Third Level]

$$\underset{x_5,x_6}{Max}\ F_3(x) = \underset{x_5,x_6}{Max}\ \sum_{k=1}^{m} \frac{b_{3j}^T x_j + \alpha_{3j}}{d_{3j}^T x + \beta_3}\ (j = 1, 2, \dots, N_3) \tag{3}$$

Where $x_7, \ldots, x_m$ solves
Subject to

$$x \in G. \quad (4)$$

Where

$$\begin{aligned} G = \{ & pr(a_{01}x_1 + a_{02}x_2 \quad + a_{0m}x_m \leq c_0) \geq \alpha_1 \\ & pr(d_1x_1 \quad \leq c_1) \geq \alpha_2, \\ & pr(d_2x_2 \quad \leq c_2) \geq \alpha_3, \\ & pr(d_mx_m \leq c_m) \geq \alpha_m, \\ & x_1, \ldots, x_m \geq 0\}. \end{aligned}$$

Where the functions $F_r(x)$ are fractional objective functions defined on R^n.
In the above problem (1)-(4), $x_j \in R, (j = 1, 2, \ldots, m)$ be a real vector variables, G is the large scale linear constraint set where, $c = (c_0, \ldots, c_m)^T$ is $(m+1)$ vector, and $a_{01}, \ldots, a_{0m}, d_1, \ldots, d_m$ are constants. Therefore $F_i : R^m \to R$, $(i = 1, 2, 3)$ be the first level objective function, the second level objective function, and the third level objective function, respectively. Moreover, the first level decision maker (FLDM) has x_1, x_2 indicating the first decision level choice, the second level decision maker (SLDM)and the third level decision maker (TLDM) have x_3, x_4 and x_5, x_6 indicating the second decision level choice and the third decision level choice, respectively.
Furthermore, Pr means probability and a_i is a specified probability value. This means that the linear constraints may be violated some of the time and at most100 $(1 - \alpha_i)\%$ of the time. For the sake of simplicity, we assume that the random parameters $c_i, (1, 2, \ldots, m)$ are distributed normally with known means $E\{c_i\}$ and variances $V\{c_i\}$ and independently of each other.

Definition 1. Let G_1, G_2, G_3 be the feasible regions of the first, the second and the third level decision maker, respectively For any $(x_1, x_2 \in G_1 = \{x_1, x_2 | (x_1, \ldots, x_m) \in G\}$ given by FLDM and $(x_3, x_4 \in G_2 = \{x_3, x_4 | (x_1, \ldots, x_m) \in G\}$ given by SLDM, if the decision-making variable $(x_5, x_6 \in G_3 = \{x_5, x_6 | (x_1, \ldots, x_m) \in G\}$ is the Pareto optimal solution of the TLDM, then $(x_1, \ldots, x_m)$ is a feasible solution ofCH-MLLSFP.

Definition 2. A point $x^* \in R^m$ is a feasible solution of the CH-MLLSFP with probability $\prod_{i=1}^{m} \alpha_i$; no other feasible solution $x \in G$ exists, such that $F_1(x^*) \leq F_1(x)$; so x^* is the Pareto optimal solution of the CH-MLLSFP.
The basic idea in treating problem(CH-MLLSFP) is to convert the probabilistic nature of this problem into an equivalent deterministic. Here, the idea of employing deterministic version will be illustrated by using the interesting technique of chance-constrained programming ([1],[10]). In this case, the set of constraints X can be rewritten in the deterministic form as:

$$G = \{X \in R^n | \sum_{j=i}^{n} a_{ij}x_j \leq E(c_i) + K_{\alpha_i}\sqrt{Var(c_i)}, i = 1, 2, \ldots, m, x_j \geq 0, j = 1, 2, \ldots, n\} \quad (5)$$

where K_{α_i} is the standard normal value such that $\Phi(K_{\alpha_i}) = 1 - \alpha_i$; and $\Phi(a)$ represents the cumulative distribution function of the standard normal distribution evaluated at a.

3 Deterministic Multilevel Large Scale Fractional Programming Problem

Now before we go any further, problem (CH-MLLSFP) can be understood as the following deterministic multi-level large scale fractional programming problem (MLLSFP):
[First Level]

$$\underset{x_1, x_2}{Max}\ F_1(x) = \underset{x_1, x_2}{Max}\ \sum_{j=1}^{m} \frac{b_{1j}^T x_j + \alpha_{1j}}{d_{1j}^T x + \beta_1},\ (j = 1, 2, \ldots, N_1) \quad (6)$$

Where $x_3, \ldots, x_m$ solves
[Second Level]

$$\underset{x_3,x_4}{Max}\ F_2(x) = \underset{x_3,x_4}{Max}\ \sum_{j=1}^{m} \frac{b_{2j}^T x_j + \alpha_{2j}}{d_{2j}^T x + \beta_2},\ (j = 1, 2, \ldots, N_2) \tag{7}$$

Where $x_5, \ldots, x_m$ solves
[Third Level]

$$\underset{x_5,x_6}{Max}\ F_3(x) = \underset{x_5,x_6}{Max}\ \sum_{j=1}^{m} \frac{b_{3j}^T x_j + \alpha_{3j}}{d_{3j}^T x + \beta_3},\ (j = 1, 2, \ldots, N_3) \tag{8}$$

Where $x_7, \ldots, x_m$ solves
Subject to:

$$G = \{X \in R^n | \sum_{j=i}^{n} a_{ij} x_j \leq E(c_i) + K_{\alpha_i} \sqrt{Var(c_i)}, i = 1, 2, \ldots, m, x_j \geq 0, j = 1, 2, \ldots, n\}$$

4 Taylor Series Approach For Multi-level Large Scale Fractional Programming Problem (MLLSFPP)

In the deterministic multilevel large scale fractional programming problem (MLLSFPP), can be transform objective functions by using Taylor series at first, and then a satisfactory value for the variables of the model is obtained by solving the model, which has a single objective function. Here, Taylor series obtains polynomial objective functions which are equivalent to fractional objective functions. Then, the (MLLSFPP) can be reduced into a single objective combined with the use of the branch and bound method. In the compromised objective function, the weight of the first objective is more than the weight of the second objective and so on.
The proposed approach to solve multilevel large scale fractional programming problem (MLLSFPP)can be explained as follows:
Step: 1 Determine $x_i^* = (x_{i1}^*, \ldots, x_{1k}^*$ which is the value that is used to maximized the i^{th} objective function $F_i(x), (i = 1, 2, \ldots, k)$ where k is number of the variables.
Step: 2 Transform the objective functions $F_i(x), (j = 1, 2, \ldots, k)$ by using the following 1^{th} order Taylor series polynomial series in the following form stated in ([2], [11])as:

$$F_i(x) \cong \bar{F}_i(x) = F_i(x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial F_i(x_i^*)}{dx_j}, (j = 1, 2, \ldots, n) \tag{9}$$

Step: 3 Find the satisfactory solution by solving the reduced problem to a single objective function.
In the compromised objective function, the weight of the first objective is more than the weight of the second objective and so on.

5 A decomposition Algorithm ForChance-Constrained Large Scale Multi-Level Fractional Programming Problem (CH-MLLSFPP)

Multilevel large scale linear programming problem(MLLSLPP) is solved by adopting the leader-follower Stakelberg strategy combine with Dantzig and Wolf decomposition method ([6], [8]).One first gets the

optimal solution that is acceptable to FLDM using the decomposition method to break the large scale problem into n-sub problems that can be solved directly.
The decomposition principle is based on representing the MLLSLPP in terms of the extreme points of the sets $d_j x_j \leq c_j, x_j \geq 0, j = 1, 2, \ldots, m.$ To do so, the solution space described by each $d_j x_j \leq c_j, x_j \geq 0, j = 1, 2, \ldots, m$ must be bounded and closed .
Then by inserting the FLDM decision variable to the SLDM for him/her to seek the optimal solutionusing Dantzig and Wolf decomposition method [8], then the decomposition method break the large scale problem into n-sub problems that can be solved directly.
Finally the TLDM do the same action till he obtains the optimal solution of his problem which is the optimal solution to TLLSLPP.

Theorem 1. *The decomposition algorithm terminates in a finite number of iterations, yielding a solution of the large scale problem.*
To prove theorem 1 above, the reader is referred to [8].

5.1 The first-level decision-maker problem

The first-level decision-maker problem of the MLLSLPP is as follows:
[First Level]

$$MaxF_1(x) = Max \sum_{j=1}^{m} b_{1j} x_j, \tag{10}$$

Subject to $x \in G$.
To obtain the optimal solution of the FLDM problem; suppose that the extreme points of $d_j x_j \leq c_j, x_j \geq 0$ are defined as $\hat{x}_{jk}, k = 1, 2, 3,$ where x_j defined by:

$$x_j = \sum_{k=1}^{k_j} \beta_{jk} \hat{x}_{jk} \quad , j = 1, \ldots, m. \tag{11}$$

and $\beta_{jk} \geq 0$, for all k and $\sum_{k=1}^{k_i} \beta_{jk} = 1.$
Now, the FLDM problem in terms of the extreme points to obtain the following master problem of the FLDM are formulated as stated in [6]:

$$Max \sum_{k=1}^{k_1} b_{11} \hat{x}_{1k} \beta_{1k} + \sum_{k=1}^{k_2} b_{12} \hat{x}_{2k} \beta_{2k} + \cdots + \sum_{k=1}^{k_n} b_{1n} \hat{x}_{nk} \beta_{nk} \tag{12}$$

Subject to

$$\sum_{k=1}^{k_1} a_{01} \hat{x}_{1k} \beta_{1k} + \sum_{k=1}^{k_2} a_{02} \hat{x}_{2k} \beta_{2k} + \cdots + \sum_{k=1}^{k_n} a_{0n} \hat{x}_{nk} \beta_{nk} \leq C_0,$$

$\sum_{k=1}^{k_1} \beta_{1k} = 1,$
$\sum_{k=1}^{k_2} \beta_{2k} = 1,$
$\sum_{k=1}^{k_n} \beta_{nk} = 1, \beta_{jk} \geq 0$, for all j and k.

The new variables in the FLDM problemare β_{jk} which determined using Balinskis algorithm [12]. Once their optimal values β^*_{jk} are obtained, then the optimal solution to the original problem can be found by back substitution as follow:

$$x_j = \sum_{k=1}^{k_j} \beta^*_{jk} \hat{x}_{jk},\ j = 1, 2, 3. \tag{13}$$

It may appear that the solution of the FLDM problem requires prior determination of all extreme points $\hat{x}_{jk}$.

To solve theFLDM problem by the revised simplex method, it must determine the entering and leaving variables at each iteration. Let us start first with the entering variables.
Given C_B and B^{-1} of the current basis of the FLDM problem, then for non-basic β_{jk}:

$$z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk} \tag{14}$$

Where

$$c_{jk} = c_j \hat{x}_{jk} \text{ and } P_{jk} = \begin{bmatrix} a_j \hat{x}_{jk} \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{15}$$

Now, to decide which of the variables β_{jk} should enter the solution it must determine:

$$\hat{z}_{jk} - \hat{c}_{jk} = \min\{z_{jk} - c_{jk}\} \tag{16}$$

Consequently, if $z^*_{jk} - c^*_{jk} \leq 0$, then according to the maximization optimality condition, β^*_{jk} must enter the solution; otherwise, the optimal has been reached.

5.2 The Second-Level Decision-Maker (SLDM) Problem

Secondly, according to the mechanism of the CH-MLLSFPP, the FLDM variables x^F_1, x^F_2 should be given to the SLDM; hence, the SLDM problem can be written as follows:

$$MaxF_2(x) = Max \sum_{j=1}^{m} b_{2j} x_j, \tag{17}$$

Subject to $(x^F_1, x^F_2, \ldots, x_m) \in G$.
To obtain the optimal solution of the SLDM problem; the SLDM solves his master problem by the decomposition method [6] as the FLDM.

5.3 The Third-Level Decision-Maker (TLDM) Problem

Finally, according to the mechanism of the CH-MLLSFPP, the SLDM variables $x^F_1, x^F_2, x^S_3, x^S_4$ should be given to the TLDM; hence, the TLDM problem can be written as follows:

$$MaxF_3(x) = Max \sum_{j=1}^{m} b_{3j} x_j, \tag{18}$$

Subject to $(x^F_1, x^F_2, x^S_3, x^S_4, \ldots, x_m) \in G$.
To obtain the optimal solution of the TLDM problem; the TLDM solves his master problem by the decomposition method [8] as the FLDM and SLDM.
Now the optimal solution $(x^F_1, x^F_2, x^S_3, x^S_4, x^T_5, x^T_6, \ldots, x^T_m)$ of the TLDM is the optimal solution of the CH-MLLSFPP.

6 An Algorithm For Solving CH-MLLSFPP

A solution algorithm to solve chance-constrained multilevel large scale fractional programming problem (CH-MLLSFPP) is described in a series of steps. This algorithm overcomes the complexity nature of multilevel large scale fractional programming problem, and uses stochastic in the constraint method of multilevel optimization to facility the large scale constraints nature. Inserting the variables value of every higher level decision maker to his lower level decision maker break the difficulty faces the CH-MLLSFPP.

The suggested algorithm can be summarized in the following manner:

Step 1.

Determine the means $E\{c_i\}$ and $Var\{c_i\}(i = 1, 2, \dots m)$.go to Step 2.

Step 2.

Transform the original set of constraints X of problem (CH-MLLSFPPs) into the equivalent set of constraints G.

$$G = \{X \in R^n | \sum_{j=1}^{n} a_{ij}x_j \leq E(c_i) + k_{\alpha_i}\sqrt{Var(c_i)}, (i = 1, 2, \dots, m), x_j \geq 0, (j = 1, 2, \dots, m)\}$$

Step 3.

Formulate the equivalent problem (MLLSFPP).

Step 4.

Convert problem (MLLSFPP) into (MLLSLPP) using Taylor series approach the transformation for the FLDM, SLDM, and TLDM.

$$F_i(x) \cong \bar{F}_i(x) = F_i(x_i^*) + \sum_{j=1}^{n}(x_i - x_{ij}^*)\frac{\partial F_i(x_i^*)}{dx_j}, (j = 1, 2, \dots, n)$$

Step 5.

Start with the FLDM problem and, go to Step 6.

Step 6.

Convert the master problem in terms of extreme points of the sets $d_jx_j \leq c_j$, $x_j \geq 0$, $j = 1, 2, 3$.

Step 7. Determine the extreme points $x_j = \sum_{k=1}^{k_j} \beta_{jk}\hat{x}_{jk}$, $j = 1, 2, 3$ using Balinski's algorithm [12].

Step 8.

Set $k = 1$.

Step 9.

Compute $z_{jk} - c_{jk} = C_B B^{-1} P_{jk} - c_{jk}$, go to Step 10.

Step 10.

If $z_{jk}^* - c_{jk}^* \leq 0$, then go to Step 11; otherwise, the optimal solution has been reached, go to Step 12.

Step 11.

Set $k = k + 1$, go to Step8.

Step 12.

If the SLDM obtain the optimal solution go to Step 15 , otherwise go to Step 13.

Step 13.

Set $(x_1, x_2) = (x_1^F, x_2^F)$ to the SLDM constraints, go to Step 14.

Step 14.

The SLDM formulate his problem, go to Step 6.

Step 15.

If the TLDM obtain the optimal solution go to Step 18, otherwise go to Step 16.

Step 16.

Set $(x_1, x_2, x_3, x_4) = (x_1^F, x_2^F, x_3^S, x_4^S)$ to the TLDM constraints, go to Step 17.

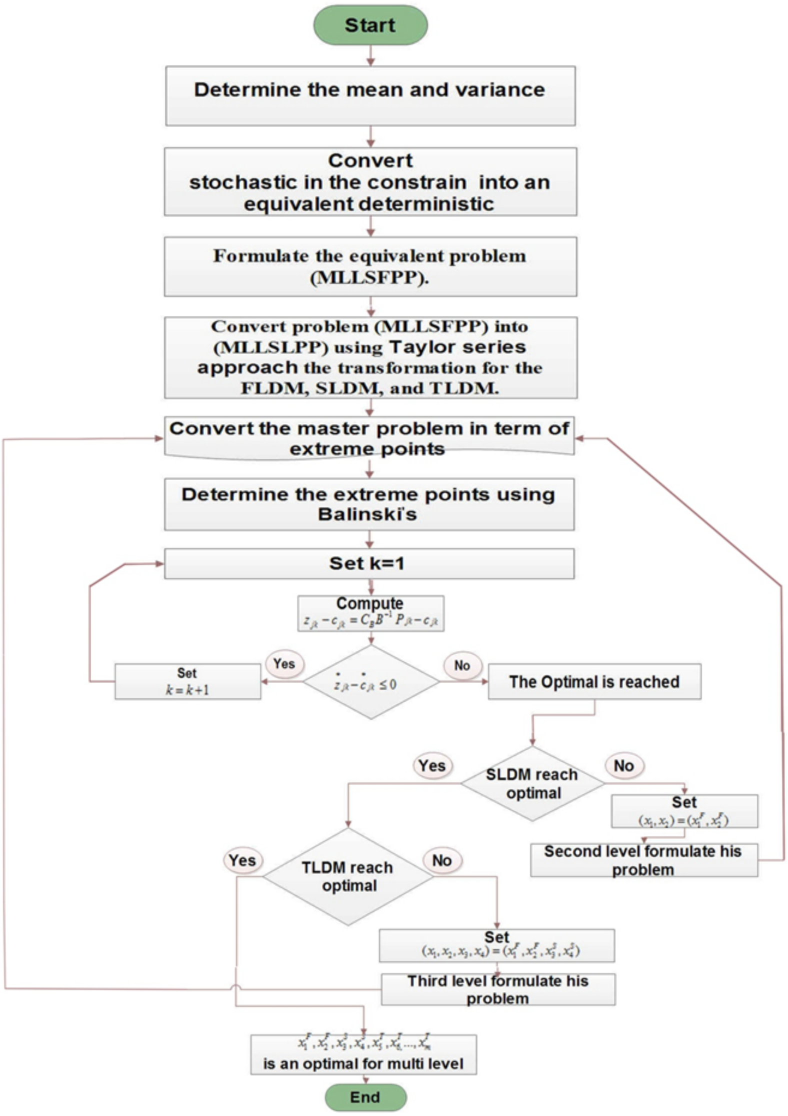
Step 17.

The TLDM formulate his problem, go to Step 6.

Step 18.

$(x_1^F, x_2^F, x_3^S, x_4^S, x_5^T, x_6^T, \ldots, x_m^T)$ is as an optimal solution for multilevel large scale linear programming problem, then stop .

7 An Algorithm Flow Chart For CH-MLLSFPP



8 An Illustrative Example

In this section, an illustrative example is given to clarify the proposed solution algorithm.To demonstrate the solution for (CH-MLLSFPP), let us consider the following problem:
[First level]

$$\underset{x_1,x_2}{Max}\ F_1(x_1,x_2) = \underset{x_1,x_2}{Max}\ \frac{2x_1+3x_2+x_5+x_6}{x_1+4x_2+6}$$

Where x_3, x_4, x_5, x_6 solves
[Second level]

$$\underset{x_3,x_4}{Max}\ F_2(x_3,x_4) = \underset{x_3,x_4}{Max}\ \frac{3x_3+4x_4+x_5+x_6}{6x_3+4x_4+2}$$

Where x_5, x_6 solves
[Third level]

$$\underset{x_5,x_6}{Max}\ F_3(x_5,x_6) = \underset{x_5,x_6}{Max}\ \frac{x_1+3x_5+2x_6}{x_5+x_6+6}$$

Subject to $Pr\{x_1+x_2+x_3+x_4+x_5+x_6 \le c_1\} \ge 0.95,$
$Pr\{2x_1+x_2 \le c_2\} \ge 0.90,$
$Pr\{x_3+2x_4 \le c_3\} \ge 0.90,$
$Pr\{x_5+\frac{1}{3}x_6 \le c_4\} \ge 0.95,$
$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$
Suppose that $bi, (i = 1, 2, 3)$ are normally distributed random parameters with the following means and variances:

Mean	$E\{c_1\} = 3.29$	$E\{c_2\} = 1.215$	$E\{c_3\} = 1.785$	$E\{c_4\} = 0.697$
Variance	$var\{c_1\} = 4$	$var\{c_2\} = 1$	$var\{c_3\} = 3$	$var\{c_4\} = 2$

From standard normal tables, we have: $K_{\alpha 2} = K_{\alpha 3} = K_{0.90} \cong 1.285, K_{\alpha 1} = K_{\alpha 4} = K_{0.95} \cong 1.645$ Problem (CH-MLLSFPP) can be represented as the following deterministic multilevel large scale fractional programming problem (MLLSFPP):
[First Level]

$$\underset{x_1,x_2}{Max}\ F_1(x_1,x_2) = \underset{x_1,x_2}{Max}\ \frac{2x_1+3x_2+x_5+x_6}{x_1+4x_2+6}$$

Where x_3, x_4, x_5, x_6 solves
[Second level]

$$\underset{x_3,x_4}{Max}\ F_2(x_3,x_4) = \underset{x_3,x_4}{Max}\ \frac{3x_3+4x_4+x_5+x_6}{6x_3+4x_4+2}$$

Where x_5, x_6 solves
[Third level]

$$\underset{x_5,x_6}{Max}\ F_3(x_5,x_6) = \underset{x_5,x_6}{Max}\ \frac{x_1+3x_5+2x_6}{x_5+x_6+6}$$

Subject to $x_1+x_2+x_3+x_4+x_5+x_6 \ge 10,$
$x_1+x_2 \le 3,$
$x_3+2x_4 \le 4,$
$x_5+\frac{1}{3}x_6 \le 3,$
$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0.$
If the problem(MLLSFP) is solved for each membership functions one by one then $h_1(4,0,2,0,1,1) = 1$, $h_2(1,0,0,2,1,1) = 1$ and $h_3(2,0,1,0,0,4) = 1$. Then, the objective functions are transformed by using 1^{th} order Taylor polynomial series.
Now, the resulting form of the problem (MLLSFP) above is written as:
[First Level]
$F_1(x) \cong \hat{F}_1(x) = 0.1x_1 - 0.1x_2 + 0.1x_5 + 0.1x_6 + 0.4,$
Where x_1, x_2 solves

[Seconed Level]
$F_2(x) \cong \hat{F}_2(x) = -0.3x_3 + 0.1x_5 + 0.1x_6 + 0.8,$
Where x_1, x_2, x_3, x_4 solves
[Third Level]
$F_3(x) \cong \hat{F}_3(x) = 0.1x_1 - 0.2x_5 + 0.1x_6 - 0.4$
Subject to
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 10,$
$x_1 + x_2 \leq 3,$
$x_3 + 2x_4 \leq 4,$
$x_5 + \frac{1}{3}x_6 \leq 3,$
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.$
Now the problem is solved by decomposition
first, the FLDM solves his/her problem as follow:
$X_B = B^{-1}(10, 1, 1, 1)^T = (1.18, 1, 1 - 0.18)^T C_B = (0, -M, -M, -M)$
After 10 iterations the FLDM obtain his optimal solution
$X_3^* = 1(0, 9)^T = (0, 9)\ \ X_1^* = 1(\frac{3}{2}, 0)^T = (1.5, 0),\ \ X_2^* = 1(4, 0)^T = (4, 0), Z_{Thebest} = 1.45,$
$(x_1^F, x_2^F, x_3^F, x_4^F, x_5^F, x_6^F) = (1.5, 0, 4, 0, 0, 9), F_1 = 1.45$
Second, the SLDM solves his/her problem as follow:
The SLDM do the same action like FLDM till he obtains the optimal solution. After 3 iterations the SLDM obtain his optimal solution. $X_1^* = 0.9(0, 9)^T = (0, 8.1), X_2^* = 0.083(3, 0)^T = (0.249, 0),$ $Z_{Thebest} = 0.825,$
$(x_1^S, x_2^S, x_3^S, x_4^S, x_5^S, x_6^S) = (1.5, 0, 0, 8.1, 0.249, 0).F_2 = 0.825.$
Third, the TLDM solves his/her problem as follows:
TLDM problem using a decomposition algorithm to solve his problem depending on
$(x_1^T, x_2^T, x_3^T, x_4^T, x_5^T, x_6^T) = (1.5, 0, 0, 8.1, 0.249, 0).$
$(x_1^F, x_2^F) = (1.5, 0)$ and $(x_3^S, x_4^S) = (0, 8.1).F_3 = 0.3.$

9 Conclusions

This paper presented a solution algorithm to solve a multi-level large scale fractional programming problem with individual chance constraints (CH-MLLSFP). The basic idea in treating (CH-MLLSFP) was to convert the probabilistic nature of this problem into a deterministic multi-level large scale fractional programming problem (MLLSFPP). A solution of multi-level large scale fractional programming problem was presented using a Taylor series to avoid the complexity of fractional nature. An illustrative example was discussed to demonstrate the correctness of the proposed solution method. However, there are many other aspects, which should by explored and studied in the area of a large scale multi-level optimization such as:

1- Multi-level large scale fractional programming problem with stochastic parameters in both objective functions and constraints.

2- Taylor series approach for solving chance constrained multi-level multi-objective mixed integer non-linear fractional programming problem.

3- Multi-level large scale programming problem with rough parameters in the objective functions and in the constraints and with integrality conditions.

Table of Symbols

CH-MLLSFP	Chance-constrained multilevel large scale fractional programming problem
MLLSFPP	Multi-level large scale fractional programming problem
MLLSLPP	Multi-level large scale linear programming problem
FLDM	The first level decision maker
SLDM	The second level decision maker
TLDM	The third level decision maker
TLLSFPP	Three level large scale fractional programming problem

Competing Interests

The authors declare that no competing interests exist.

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