British Journal of Mathematics & Computer Science
6(2): 129-145, 2015, Article no.BJMCS.2015.068
ISSN: 2231-0851

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A Comparative Study of Successive Approximations Method and He-Laplace Method

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Article Information

DOI: 10.9734/BJMCS/2015/14624
Editor(s):
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Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=732&id=6&aid=7392

Method Article

Received: 10 October 2014
Accepted: 21 November 2014
Published: 17 December 2014

Abstract

In this work, we investigate and compare the Successive approximations method and He-Laplace method for solving the some problems of partial differential equations.

Keywords: Successive approximations method, He-Laplace transform Method, He's polynomials, linear and nonlinear partial differential equations.

1 Introduction

Many important phenomena occurring in different fields of engineering, physics, biology and another science are frequently modeled through differential equations. However, it is still very difficult to get exact solutions form for most models of real life problems abroad class of analytical methods and numerical method were used to handle such problem. In recent years, many authors have attention to study the solution of linear and Nonlinear PDEs by using different methods. For example Adomain decomposition method [1-6], finite different method [7,8], variational iteration method [9-12], weighted finite difference technique [13], Laplace decomposition method [14-16], integral transform [17], He-Laplace method, Homotopy Perturbation Method, Successive approximation method, etc . This paper outlines a reliable Comparison between two powerful methods that were recently developed. The first is the

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successive approximations method (SAM) is one classical method for solving integral equations [18]. It is also called the Picard iteration method in the literature. In fact, this method provides a scheme that one can use for solving initial value problems. First to starts by finding successive approximations to the solution by writing an initial guess, called the zeroth approximation, which is any selective real-valued function that one uses in a recurrence relation to determine the other approximations [18]. The second is He-Laplace method is combination of the Laplace transformation, He's polynomials and the homotopy perturbation method. We use the homotopy perturbation method coupled with the Laplace transformation for solving some problems of linear and nonlinear PDEs.

This paper has been designed as following. Next section the methods, Then, application, lastly, conclusion.

2 The Methods

To convey first Homotopy Perturbation Method (HPM) and He's Polynomials, second the He-Laplace Transform Method, third Successive approximations Method.

2.1 Homotopy Perturbation Method (HPM) and He's Polynomials

We suppose a general equation of the form

$$\mathrm{L(u)} = 0, \tag{2.1}$$

where L is any integral . We define a convex homotopy $\mathrm{H(u,p)}$ with boundary conditions of

$$\mathrm{H(u,p)} = (1 - \mathrm{p})\mathrm{F(u)} + \mathrm{pL(u)} \tag{2.2}$$

where $\mathrm{F(u)}$ is a functional operator with known solutions $\mathrm{v_0}$.[19] It is clear that, for

$$\mathrm{H(u,p)} = 0 \tag{2.3}$$

Then we get

$$\mathrm{H(u,0)} = \mathrm{F(u)}\,, \quad \mathrm{H(u,1)} = \mathrm{L(u)}$$

This suggests that H $(\mathrm{u,p})$ continuously traces an implicitly knows curve from the beginning point H $(\mathrm{v_0},0)$ to a solution function H $(\mathrm{u},1)$[19]. The embedding parameter monotonically increases from zero to unit as the trivial problemF $(\mathrm{u}) = 0$, is continuously deforms the original problem $\mathrm{L(u)} = 0$. The embedding parameter $p \in (0,1]$ can be supposed as an expanding parameter [19-43]. The homotopy perturbation method uses the homotopy parameter p as an expanding parameter [19-27] to get

$$\mathrm{u} = \sum_{\mathrm{i}=0}^{\infty} \mathrm{P^i u_i} = \mathrm{u_0} + \mathrm{pu_1} + +\mathrm{p^2u_2} + \mathrm{p^3u_3} + \cdots \tag{2.4}$$

If $p \to 1$, then (2.4) corresponds to (2.2) and becomes the approximate solution of the form:

$$u = \lim_{p\to 1} u_n = u_0 + u_1 + u_2 + u_3 + \cdots \tag{2.5}$$

It is well known that series (2.5) is convergent for most of the cases and also the rate of convergence is Relies on $L(u)$; [19-27]. We suppose that (2.5) has a unique solution. The comparisons of like powers of p give solutions of different orders. In sum, according to [28, 29], He's HPM supposes the nonlinear term $N(u)$ as:

$$N(u) = \sum_{i=0}^{\infty} p^i H_i = H_0 + pH_1 + p^2 H_2 + \cdots, \tag{2.6}$$

where H_n's are the so-called He's polynomials [28,29], which can be calculated by using the form

$$H_n(u_0, \cdots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{n} p^i u_i\right)\right]_{p=0} \quad n = 0,1,2,\cdots \tag{2.7}$$

2.2 He-Laplace Method

We suppose a general nonlinear nonhomogeneous Partial differential equation with initial conditions [19]

$$\frac{\partial^2 y}{\partial t^2} + R_1 y(x,t) + R_2 y(x,t) + Ny(x,t) = f(x,t) \tag{2.8}$$
$$y(x,0) = \alpha(x), \qquad \frac{\partial y}{\partial t}(x,0) = \beta(x),$$

where $R_1 = \partial^2/\partial x^2$ and $R_2 = \partial/\partial x$ are the linear differential operators, N represents the general nonlinear differential operator and $f(x,t)$ is the source term[19]. Applying the Laplace transform (denoted by L) on both sides of (2.8) we get

$$L\left[\frac{\partial^2 y}{\partial t^2}\right] + L[R_1 y(x,t) + R_2 y(x,t)] + L[Ny(x,t)] = L[f(x,t)]$$
$$\Rightarrow \; s^2 L[y(x,t)] - sy(x,0) - \frac{\partial y}{\partial t}(x,0) \tag{2.9}$$
$$= -L[R_1 y(x,t) + R_2 y(x,t)] - L[Ny(x,t)] + L[f(x,t)].$$

Applying the initial conditions given in (2.8), we have

$$L[y(x,t)] = \frac{\alpha(x)}{s} + \frac{\beta(x)}{s^2} - \frac{1}{s^2}\left(L[R_1 y(x,t) + R_2 y(x,t)] - L[Ny(x,t)]\right) + \frac{1}{s^2}\left(L[f(x,t)]\right) \tag{2.10}$$

Operating the inverse Laplace transform of (2.10), we get

$$y(x,t) = F(x,t) - L^{-1}\left[\frac{1}{s^2}\left(L[R_1 y(x,t) + R_2 y(x,t)] - L[Ny(x,t)]\right)\right], \tag{2.11}$$

where $F(x, t)$ represents the term arising from the source term and the prescribed initial conditions. We apply the homotopy perturbation method:

$$y(x,t) = \sum_{n=0}^{\infty} p^n y_n(x,t), \tag{2.12}$$

and the nonlinear term can be decomposed as

$$Ny(x,t) = \sum_{n=0}^{\infty} p^n H_n(y). \tag{2.13}$$

For some He's polynomials H_n(see [44,45]) with the coupling of the Laplace transform and the homotopy perturbation method are given by

$$\sum_{n=0}^{\infty} p^n y_n(x,t) = F(x,t) - p\left(L^{-1}\left[\frac{1}{s^2}L\left[(R_1+R_2)\sum_{n=0}^{\infty} p^n y_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(y)\right]\right]\right). \tag{2.14}$$

Comparing the coefficients of same powers of p, on (2.14) we have the following approximations

$$\begin{aligned}
&p^0: \quad y_0(x,t) = F(x,t),\\
&p^1: y_1(x,t) = -L^{-1}\left(\frac{1}{s^2}L[(R_1+R_2)y_0(x,t)] + H_0(y)]\right),\\
&p^2: y_2(x,t) = -L^{-1}\left(\frac{1}{s^2}L[(R_1+R_2)y_1(x,t)] + H_1(y)]\right),\\
&p^3: y_3(x,t) = -L^{-1}\left(\frac{1}{s^2}L[(R_1+R_2)y_2(x,t)] + H_2(y)]\right),\\
&\vdots
\end{aligned} \tag{2.15}$$

2.3 Successive Approximation Method (SAM)

To convey the basic idea of the successive approximations method (SAM), we suppose the following general nonlinear differential equation:

$$L[u(t)] + R[u(t)] + N[u(t)] = K(t), \qquad t > 0. \tag{2.16}$$

where $L = \frac{d^m}{dt^m}$, $m \in N$ is the highest order derivative, $R[u(t)]$ is the reminder linear term, $N[u(t)]$ is a nonlinear operator and $K(t)$ is the inhomogeneous source term, subject to the initial conditions

$$u^{(k)}(0) = c_k \quad , k = 0,1,2, \dots, m-1 \tag{2.17}$$

We are looking for a solution u of (2.16). We shall suppose that (2.16) admits a unique solution. Otherwise, the SAM will give a solution between many (possible) other solution.

The successive approximations method considers the approximate solution of an integral equation a sequence usually converging to the accurate solution [18]. For solving Eq. (2.16) using SAM we apply $L^{-1}[.]$, which is

$$L^{-1}[.] = \frac{1}{(m-1)!}\int_0^t (t-\tau)^{m-1}[.]d\tau \tag{2.18}$$

on both sides of (2.16) so that we get

$$u(t) = \sum_{k=0}^{m-1} c_k \frac{t^k}{k!} + L^{-1}\big(k(t)\big) - L^{-1}(R[u(t)]) - L^{-1}(N[u(t)]) \tag{2.19}$$

The SAM consists of representing the solution of (2.19) as a sequence

$$\{u_n(x)\}_{n=0}^{\infty} \tag{2.20}$$

The method introduces the recurrence relation

$$u_{n+1}(t) = \sum_{k=0}^{m-1} c_k \frac{t^k}{k!} + L^{-1}\big(k(t)\big) - L^{-1}(R[u_n(t)]) - L^{-1}(N[u_n(t)]) \tag{2.21}$$

where the zeroth approximation $u_0(x)$ is an arbitrary real function. Several successive approximations $u_n, n \geq 1$ will be determined as

$$\begin{aligned} u_1(t) &= \sum_{k=0}^{m-1} c_k \frac{t^k}{k!} + L^{-1}\big(k(t)\big) - L^{-1}(R[u_0(t)]) - L^{-1}(N[u_0(t)]) \\ u_2(t) &= \sum_{k=0}^{m-1} c_k \frac{t^k}{k!} + L^{-1}\big(k(t)\big) - L^{-1}(R[u_1(t)]) - L^{-1}(N[u_1(t)]) \\ &\vdots \end{aligned} \tag{2.22}$$

$$u_{n+1}(t) = \sum_{k=0}^{m-1} c_k \frac{t^k}{k!} + L^{-1}\big(k(t)\big) - L^{-1}(R[u_n(t)]) - L^{-1}(N[u_n(t)]) \tag{2.23}$$

and the solution computed as:

$$u(x) = \lim_{n\to\infty} u_n(x) \tag{2.24}$$

The SAM is very simple in its principles. The difficulties consist in proving the convergence of the introduced series. For convergence of this method we refer the reader to [18].

3 Applications

Example 3.1 [19], [46]: Suppose the following

$$\frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2} = 0 \tag{3.1}$$

with the following conditions:

$$y(x,0) = e^x - x, \qquad y(0,t) = 1 + t, \quad \frac{\partial y}{\partial x}(1,t) = e - 1 \tag{3.2}$$

3.1.1. Using He-Laplace method

Applying the He- Laplace method of both sides of equations (3.1) & use the conditions (3.2), we get

$$y(x,s) = \frac{(e^x - x)}{s} - \frac{1}{s} L\left[\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right] \tag{3.3}$$

The inverse of the Laplace transform of (3.3) implies that

$$y(x,t) = e^x - x - L^{-1}\left[\frac{1}{s}\left[\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right]\right] \tag{3.4}$$

We apply the homotopy perturbation method both sides of (3.4); we have

$$\sum_{n=0}^{\infty} p^n y_n(x,t) = e^x - x - P\left(L^{-1}\left[\frac{1}{s}\left[\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2}\right]\right]\right), \tag{3.5}$$

Comparing the coefficient of same powers of p, on (3.5) we get

$$\begin{aligned} &p^0: y_0(x,t) = e^x - x,\\ &p^1: y_1(x,t) = -L^{-1}\left[\frac{1}{s} L\left[\frac{\partial y_0}{\partial x} - \frac{\partial^2 y_0}{\partial x^2}\right]\right] = t,\\ &p^2: y_2(x,t) = -L^{-1}\left[\frac{1}{s} L\left[\frac{\partial y_1}{\partial x} - \frac{\partial^2 y_1}{\partial x^2}\right]\right] = 0, \end{aligned} \tag{3.6}$$

We have

$$\begin{aligned} &p^3: y_3(x,t) = 0,\\ &p^4: y_4(x,t) = 0,\\ &\quad\vdots \end{aligned} \tag{3.7}$$

The solution is

$$\begin{aligned} y(x,t) &= e^x - x + t + 0 + 0 + \cdots\\ &= e^x - x + t. \end{aligned} \tag{3.8}$$

The exact solution of (3.1) & (3.2).

3.1.2 The successive approximations method

Equation (3.1) can be write as

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x} \tag{3.9}$$

We applying $L^{-1}[\cdot]$ on both sides of (3.9),

$$y(x,t) = y(x,0) + \int_0^t \left[\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x}\right] d\tau. \tag{3.10}$$

For the zeroth approximation $y_0(t)$, we can select $y_0(x,t) = 0$. then the equation (3.10) we have the iteration formula

$$y_{n+1}(x,t) = e^x - x + \int_0^t \left[\frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial x}\right] d\tau. \tag{3.11}$$

Substituting $y_0(x,t) = 0$ into (3.11) we obtain

$$\begin{aligned} &y_1(x,t) = e^x - x, \\ &y_2(x,t) = e^x - x + \int_0^t \left[\frac{\partial^2}{\partial x^2}(e^x - x) - \frac{\partial}{\partial x}(e^x - x)\right] d\tau = e^x - x + t, \qquad (3.12) \\ &y_3(x,t) = e^x - x + \int_0^t \left[\frac{\partial^2}{\partial x^2}(e^x - x + t) - \frac{\partial}{\partial x}(e^x - x + t)\right] d\tau = e^x - x + t, \\ &\vdots \end{aligned}$$

The solution $y(x,t)$ is given by

$$\begin{aligned} &y(x,t) = y_0(x,t) + y_1(x,t) + y_2(x,t) + y_3(x,t) + \cdots \\ &y(x,t) = e^x - x + t. \end{aligned} \tag{3.13}$$

which is exact solution of (3.1) & (3.2).

Example 3.2 [19], [46]: Suppose the following

$$\frac{\partial^2 y}{\partial t^2} + y - \frac{\partial^2 y}{\partial x^2} = 0, \tag{3.14}$$

with the following conditions:

$$y(x,0) = e^{-x} + x, \quad \frac{\partial y}{\partial t}(x,0) = 0 \tag{3.15}$$

3.2.1 Using He-Laplace method

Applying the He- Laplace method of both sides of equations (3.14) & use the conditions (3.15), we get

$$y(x,s) = \frac{e^{-x} + x}{s} - \frac{1}{s^2} L\left[y - \frac{\partial^2 y}{\partial x^2}\right] \tag{3.16}$$

The inverse of the Laplace transform of (3.16) implies that

$$y(x,t) = e^{-x} + x - L^{-1}\left[\frac{1}{s^2} L\left[y - \frac{\partial^2 y}{\partial x^2}\right]\right] \tag{3.17}$$

we apply the homotopy perturbation method both sides of (3.17); we have

$$\sum_{n=0}^{\infty} p^n y_n(x,t) = e^{-x} + x - p\left(L^{-1}\left[\frac{1}{s^2} L\left[y - \frac{\partial^2 y}{\partial x^2}\right]\right]\right). \tag{3.18}$$

Comparing the coefficient of same powers of p, on (3.18) we have

$$\begin{aligned} &p^0: y_0(x,t) = e^{-x} + x, \\ &p^1: y_1(x,t) = -L^{-1}\left[\frac{1}{s^2} L\left[y_0 - \frac{\partial^2 y_0}{\partial x^2}\right]\right] = \frac{-xt^2}{2!}, \\ &p^2: y_2(x,t) = -L^{-1}\left[\frac{1}{s^2} L\left[y_1 - \frac{\partial^2 y_1}{\partial x^2}\right]\right] = \frac{xt^4}{4!}, \end{aligned} \tag{3.19}$$

We have

$$\begin{aligned} &p^3: y_3(x,t) = \frac{-xt^6}{6!} \\ &p^4: y_4(x,t) = \frac{xt^8}{8!}, \\ &\quad \vdots \\ &p^n: y_n(x,t) = \frac{(-1)^n xt^{2n}}{2n!}, \end{aligned} \tag{3.20}$$

The solution $y(x,t)$ is

$$\begin{aligned} y(x,t) &= y_0 + y_1 + y_2 + y_3 + \cdots \\ &= e^{-x} + x - \frac{xt^2}{2!} + \frac{xt^4}{4!} - \frac{xt^6}{6!} + \cdots + \frac{(-1)^n xt^{2n}}{2n!} \\ &= e^{-x} + x\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \cdots + \frac{(-1)^n\, t^{2n}}{2n!}\right) \\ &= e^{-x} + x\,\mathrm{Cos}(t), \end{aligned} \tag{3.21}$$

the exact solution of (3.14) & (3.15).

3.2.2 The successive approximations method

Equation (3.14) can be write as

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} - y \tag{3.22}$$

We applying $L^{-1}[\cdot]$ on both sides of (3.22),

$$y(x,t) = y(x,0) + \int_0^t \int_0^t \left[\frac{\partial^2 y}{\partial x^2} - y\right] d\tau \tag{3.23}$$

For the zeroth approximation $y_0(t)$, we can select $y_0(x,t) = 0$. then the equation (3.23) we have the iteration formula

$$y_{n+1}(x,t) = e^{-x} + x + \int_0^t \left[\frac{\partial^2 y}{\partial x^2} - y\right] d\tau \tag{3.24}$$

Substituting $y_0(x,t) = 0$ into (3.24) we obtain

$$y_1(x,t) = e^{-x} + x$$

$$y_2(x,t) = e^{-x} + x + \int_0^t \int_0^t \left[\frac{\partial^2}{\partial x^2}(e^{-x} + x) - (e^{-x} + x)\right] dt = e^{-x} + x - x\frac{t^2}{2}, \tag{3.25}$$

$$y_3(x,t) = e^{-x} + \int_0^t \int_0^t \left[\frac{\partial^2}{\partial x^2}\left(e^{-x} + x - x\frac{t^2}{2}\right) - (e^{-x} + x - x\frac{t^2}{2})\right] dt = e^{-x} + x - x\frac{t^2}{2} + x\frac{t^4}{4!},$$

And so on

The solution $y(x,t)$ is

$$y(x,t) = y_0(x,t) + y_1(x,t) + y_2(x,t) + y_3(x,t) + \cdots$$

$$y(x,t) = e^{-x} + x - x\frac{t^2}{2} + x\frac{t^4}{4!} - x\frac{t^6}{6!} + x\frac{t^8}{8!} - \cdots \tag{3.26}$$

$$y(x,t) = e^{-x} + x\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!} - \cdots + \frac{(-1)^n t^{2n}}{2n!}\right)$$

$$y(x,t) = e^{-x} + x\mathrm{Cos}(t).$$

The exact solution of the (3.14) & (3.15).

Example 3.3 [19], [34]: Suppose the following

$$\frac{\partial y}{\partial t} - y\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2} = 0, \tag{3.27}$$

which the following conditions:

$$y(x,0) = 1 - x, \quad y(0,t) = \frac{1}{(1+t)}, \qquad y(1,t) = 0. \tag{3.28}$$

3.3.1 Using He-Laplace method

Applying the He- Laplace method of both sides the equations (3.27) & use the condition (3.28), we get

$$y(x,s) = \frac{1-x}{s} + \frac{1}{s}L\left[\frac{\partial^2 y}{\partial x^2} + y\frac{\partial y}{\partial x}\right] \tag{3.29}$$

The inverse of the Laplace transform of (3.29) implies that

$$y(x,s) = 1 - x + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 y}{\partial x^2} + y\frac{\partial y}{\partial x}\right]\right]. \tag{3.30}$$

we apply the homotopy perturbation method both sides of (3.17); we get

$$\sum_{n=0}^{\infty} p^n y_n(x,t) = 1 - x + p\left(L^{-1}\left[\frac{1}{s}\left\{L\left[\frac{\partial^2 y}{\partial x^2}\right] + L\left[\sum_{n=0}^{\infty} p^n H_n(y)\right]\right\}\right]\right), \tag{3.31}$$

where $H_n(y)$ are He's polynomials. The first few components of He's polynomials are given by

$$\begin{aligned} H_0(y) &= y_0 \frac{\partial y_0}{\partial x} = -(1-x), \\ H_1(y) &= y_0 \frac{\partial y_1}{\partial x} + y_1 \frac{\partial y_0}{\partial x} = 2(1-x)t, \\ H_2(y) &= y_0 \frac{\partial y_2}{\partial x} + y_1 \frac{\partial y_1}{\partial x} + y_2 \frac{\partial y_0}{\partial x} = -3(1-x)t^2. \\ &\vdots \end{aligned} \tag{3.32}$$

Comparing the coefficient of same powers of p, on (3.31) we have

$$\begin{aligned} p^0 &: y_0(x,t) = 1 - x, \\ p^1 &: y_1(x,t) = -L^{-1}\left[\frac{1}{s}\left\{L\left[\frac{\partial^2 y_0}{\partial x^2}\right] + L[H_0(y)]\right\}\right] = -(1-x)t, \\ p^2 &: y_2(x,t) = -L^{-1}\left[\frac{1}{s}\left\{L\left[\frac{\partial^2 y_1}{\partial x^2}\right] + L[H_1(y)]\right\}\right] = (1-x)t^2, \end{aligned} \tag{3.33}$$

We have

$$\begin{aligned} p^3 &: y_3(x,t) = -(1-x)t^3, \\ p^4 &: y_4(x,t) = (1-x)t^4 \\ &\vdots \end{aligned} \tag{3.34}$$

The solution $y(x,t)$ is

$$\begin{aligned} y(x,t) &= y_0 + y_1 + y_2 + y_3 + \cdots \\ &= (1-x)-(1-x)t+(1-x)+t^2 + (1-x)t^3 + \cdots \\ &= (1-x)\,[\,1 - t + t^2 - t^3 + t^4 - \cdots] \\ &= (1-x)(1+t)^{-1} = \frac{(1-x)}{(1+t)}, \end{aligned} \tag{3.35}$$

The exact solution of the (3.27) & (3.28).

3.3.2 The successive approximations method

Equation (3.27) can be write as

$$\frac{\partial y}{\partial t} = y\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}, \tag{3.36}$$

We applying $L^{-1}[\cdot]$ on both sides of (3.36),

$$y(x,t) = y(x,0) + \int_0^t \left[y\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}\right] d\tau \tag{3.37}$$

For the zeroth approximation $y_0(t)$, we can select $y_0(x,t) = 0$. Then the equation (3.37) we have the iteration formula

$$y_{n+1}(x,t) = 1 - x + \int_0^t \left[y\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2} \right] d\tau \tag{3.38}$$

Substituting $y_0(x,t) = 0$ into (3.38) we get

$$y_1(x,t) = 1 - x,$$

$$y_1(x,t) = 1 - x + \int_0^t \left[(1-x)\frac{\partial}{\partial x}(1-x) + \frac{\partial^2}{\partial x^2}(1-x) \right] d\tau = (1-x) - (1-x)t, \tag{3.39}$$

$$y_2(x,t) = 1 - x + \int_0^t \left[(1-x-t+xt)\frac{\partial}{\partial x}(1-x-t+xt) + \frac{\partial^2}{\partial x^2}(1-x-t-xt) \right] d\tau$$

$$= (1-x) - (1-x)t - (1-x)t^2 - (1-x)\frac{t^3}{3},$$

$$\vdots$$

The solution $y(x,t)$ is

$$y(x,t) = y_0 + y_1 + y_2 + y_3 + \cdots$$

$$= (1-x)\left(1 - t - t^2 - \frac{t^3}{3} - \cdots\right). \tag{3.40}$$

the exact solution of (3.27) & (3.28).

Example 3.4 (34): Suppose the following

$$\frac{\partial y}{\partial t} - y - y\frac{\partial^2 y}{\partial x^2} - \left(\frac{\partial y}{\partial x}\right)^2 = 0 \qquad (3.41),$$

with the following conditions:

$$y(x,0) = \sqrt{x}, \quad y(0,t) = 0, \quad y(1,t) = e^t. \tag{3.42}$$

<u>3.4.1 Using He-Laplace method</u>

Applying the He- Laplace method of both sides the equations (3.41) & use the condition (3.42), we get

$$y(x,s) = \frac{\sqrt{x}}{s} + \frac{1}{s}L\left[y + y\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 \right]. \tag{3.43}$$

The inverse of the Laplace transform of (3.43) implies that

$$y(x,t) = \sqrt{x} + L^{-1}\left[\frac{1}{s}L\left[y + y\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 \right] \right]. \tag{3.44}$$

we apply the homotopy perturbation method of (3.44), we get

$$\sum_{n=0}^{\infty} p^n y_n(x,t) = \sqrt{x} + p\left(L^{-1}\left[\frac{1}{s}\left\{ L[y] + L\left[\sum_{n=0}^{\infty} p^n H_n(y) \right] \right\} \right] \right) \tag{3.45}$$

where $H_n(y)$ are He's polynomials. The first few components of He's polynomials are given by

$$\begin{aligned} H_0(y) &= y_0 \frac{\partial^2 y_0}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 = 0 \\ H_1(y) &= y_0 \frac{\partial^2 y_1}{\partial x^2} + y_1 \frac{\partial^2 y_0}{\partial x^2} + 2\frac{\partial y_0}{\partial x}\frac{\partial y_1}{\partial x} = 0 \\ H_2(y) &= y_0 \frac{\partial^2 y_2}{\partial x^2} + y_1 \frac{\partial^2 y_1}{\partial x^2} + y_2 \frac{\partial^2 y_0}{\partial x^2} + \left(\frac{\partial y_1}{\partial x}\right)^2 + 2\frac{\partial y_0}{\partial x}\frac{\partial y_2}{\partial x} = 0 \\ &\vdots \end{aligned} \tag{3.46}$$

Comparing the coefficient of like powers ofp, on (3.46) we have

$$\begin{aligned} p^0 &: y_0(x,t) = \sqrt{x}, \\ p^1 &: y_1(x,t) = L^{-1}\left[\frac{1}{s}\{L[y_0] + L[H_0(y)]\}\right] = \sqrt{x}\,t, \\ p^2 &: y_2(x,t) = L^{-1}\left[\frac{1}{s}\{L[y_1] + L[H_1(y)]\}\right] = \frac{\sqrt{x}\,t^2}{2!}. \end{aligned} \tag{3.47}$$

We have

$$\begin{aligned} p^3 &: y_3(x,t) = \frac{\sqrt{x}\,t^3}{3!}, \\ p^4 &: y_4(x,t) = \frac{\sqrt{x}\,t^4}{4!}, \\ &\vdots \end{aligned} \tag{3.48}$$

The solution $y(x,t)$ is

$$\begin{aligned} y(x,t) &= y_0 + y_1 + y_2 + y_3 + \cdots \\ &= \sqrt{x} + \frac{\sqrt{x}\,t}{1!} + \frac{\sqrt{x}\,t^2}{2!} + \frac{\sqrt{x}\,t^3}{3!} + \cdots \\ &= \sqrt{x}\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots + \frac{t^n}{n!} + \cdots\right) \\ &= \sqrt{x}\,e^t, \end{aligned} \tag{3.49}$$

The exact solution of (3.41) & (3.42).

<u>4.3.2 The successive approximations method</u>

Equation (3.41) can be write as

$$\frac{\partial y}{\partial t} = y + y\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 \tag{3.50}$$

We applying $L^{-1}[\cdot]$ on both sides of (3.50),

$$y(x,t) = y(x,0) + \int_0^t \left[y + y\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2\right] d\tau \tag{3.51}$$

For the zeroth approximation $y_0(t)$, we can select $y_0(x,t) = 0$. then the equation (3.37) we have the iteration form

$$y_n(x,t) = \sqrt{x} + \int_0^t \left[y + y\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 \right] d\tau \tag{3.52}$$

Substituting $y_0(x,t) = 0$ into (3.52) we get

$$y_1(x,t) = \sqrt{x}\,,$$

$$y_2(x,t) = \sqrt{x} + \int_0^t \left[\sqrt{x} + (\sqrt{x})\frac{\partial^2}{\partial x^2}(\sqrt{x}) + \frac{\partial}{\partial x}(\sqrt{x})\right] d\tau = \sqrt{x} + \sqrt{x}\,t,$$

$$y_3(x,t) = \sqrt{x} + \int_0^t \left[(\sqrt{x} + \sqrt{x}\,t) + (\sqrt{x} + \sqrt{x}\,t)\frac{\partial^2}{\partial x^2}(\sqrt{x} + \sqrt{x}\,t) + \frac{\partial}{\partial x}(\sqrt{x} + \sqrt{x}\,t)\right] d\tau$$
$$= \sqrt{x} + \sqrt{x}\,\frac{t^2}{2!},$$

So that the solution $y(x,t)$ is given by

$$y(x,t) = y_0 + y_1 + y_2 + y_3 + \cdots$$

$$y(x,t) = \sqrt{x} + \sqrt{x}\,t + \sqrt{x}\frac{t^2}{2!} + \sqrt{x}\frac{t^3}{3!} + \cdots + \sqrt{x}\,\frac{t^n}{n!} + \cdots$$

$$y(x,t) = \sqrt{x}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots + \frac{t^n}{n!} + \cdots\right)$$
$$y(x,t) = \sqrt{x}\,e^t,$$

The exact solution of (3.41) & (3.42).

4 Conclusions and Discussions

The main goal of this work is to conduct a comparative study between the successive approximations method and the He-Laplace method. The two methods are powerful and efficient methods that both give approximations of higher accuracy and closed form solutions if existing.

An important conclusion can made here .The successive approximations method for solving the some problems of partial differential equations. The same problems are solved by He-Laplace method. However, He-Laplace is combination of the Laplace transformation, the homotopy perturbation method and He's polynomials. Moreover, He-Laplace method. provides the components of exact solution, The He-Laplace is capable of reducing the volume of the computational works as compared to the classical methods while still maintaining the high accuracy of the numerical results.

Acknowledgments

The author would like to thank the referee for their helpful comment.

Competing Interests

Author has declared that no competing interests exist.

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